e-ISSN 2540-9840 p-ISSN 1412-3770 Jurnal Ilmiah Sains, April 2023, 23(1): 40-46
Accredited by Ministry of Education, Culture, Research and Technology No: 105/E/KPT/2022
DOI: https://doi.org/10.35799/jis.v23i1.46771
Available online at https://ejournal.unsrat.ac.id/v3/index.php/jis

Odd Harmonious Labeling of the Zinnia Flower Graphs

Fery Firmansah[1*], Tasari[2], Muhammad Ridlo Yuwono[3]
[1, 2, 3]**Department of Mathematics Education, Widya Dharma University, Klaten, Indonesia**
*Corresponding author: feryfirmansah@unwidha.ac.id.

ABSTRACT

An odd harmonious graph is a graph that satisfies the odd harmonious labeling properties. In this study, a new graph class construction is presented, namely zinnia flower graphs and variations of the zinnia flower graphs. The research method used is qualitative and includes several phases, namely data collection, data processing and analysis, and verification of the results. The purpose of this research is to find a new class of graphs that is a family of odd harmonious graphs. We will prove that prove that the zinnia flower graph and its variations satisfy odd harmonious labeling properties. The result of this research is that the zinnia flower graph and its variations are odd harmonious graphs.

Keywords: Flower graph; odd harmonious graph; odd harmonious labelling; zinnia flower graph

Pelabelan Harmonis Ganjil dari Graf Bunga Zinnia

ABSTRAK

Graf yang memiliki sifat pelabelan harmonis ganjil adalah graf harmonis ganjil. Pada penelitian ini akan didapatkan konstruksi graf bunga zinnia dan variasi graf bunga zinnia. Metode penelitian yang digunakan adalah penelitian kualitatif yang terdiri beberapa tahapan yaitu pengumpulan data, pengolahan dan analisis data, serta verifikasi hasil. Tujuan penelitian ini adalah menemukan kelas graf baru yang merupakan keluarga dari graf harmonis ganjil. Hasil penelitian ini diperoleh bahwa graf bunga zinnia dan variasi graf bunga zinnia merupakan graf harmonis ganjil.

Kata kunci: graf bunga; graf bunga zinnia; graf harmonis ganjil; pelabelan harmonis ganjil

(Article History: Received 28-02-2023; Accepted 29-04-2023; Published 31-04-2023)

INTRODUCTION

The topic of research on graph labeling has grown tremendously in recent years, as evidenced by the various types of research results on graph labeling (Gallian, 2022). One of the research topics on graph labeling is odd harmonious graph labeling. Liang and Bai introduced odd harmonious graphs in 2009. Graph $G(p,q)$ is odd harmonious if the vertex labeling function is injective $g: V(G) \rightarrow \{0,1,2,3,4,\ldots,2q-1\}$ and the edge labeling function is bijective with $g^*: E(G) \rightarrow \{1,3,5,7,9,\ldots,2q-1\}$ with $g^*(mn) = g(m) + g(n)$ (Liang & Bai, 2009). In the same paper, Liang and Bai proved that cycle graphs, complete graphs, bipartite graphs, and windmill graphs are odd harmonious graphs.

In a different paper, Abdel-All and Seoud (2016) also found a class of odd harmonious graphs (Abdel-Aal & Seoud, 2016). Jeyanti et al in 2015 also found several classes of odd harmonious graphs (Jeyanthi et al., 2015). Other relevant research results are as follows (Abdel-Aal, 2013), (Firmansah & Yuwono, 2017a), (Firmansah, 2017), (Firmansah & Yuwono, 2017b), (Seoud & Hafez, 2018), and (Jeyanthi & Philo, 2019).

In 2020 Febriana and Sugeng proved that odd harmonious labeling on squid graphs (Febriana & Sugeng, 2020). In 2021 Sarasvati et al proved that edge combination product are odd harmonious graphs (Sarasvati et al., 2021). Firmansah proved that multiple net snake graphs are odd harmonious graphs (Firmansah, 2020b). In a different paper, results of other relevant research in 2020, 2021 and 2022 are as follows (Firmansah, 2020a), (Firmansah & Tasari, 2020), (Firmansah & Giyarti, 2021), (Philo & Jeyanthi, 2021), (Firmansah, 2022), and (Firmansah, 2023).

The purpose of this research is to find a new class of graphs that is a family of odd harmonious graphs. In this paper, we will construct new graph classes are the definition of the zinnia flower graph $Z(h)$ with $h \geq 1$ and variations of the zinnia flower graph $Z_v(h)$ with $h \geq 1$. Furthermore, it will be proved that zinnia flower graph $Z(h)$ with $h \geq 1$ and variations of the zinnia flower graph $Z_v(h)$ with $h \geq 1$ satisfy the properties of odd harmonious labeling.

RESEARCH METHOD

The research in this paper is of the qualitative research type. The research stages consist of data collection, data processing and analysis, and verification of results. The data collection stage consists of collecting the latest research results on new graph construction zinnia flower graph $Z(h)$ with $h \geq 1$ and variations of the zinnia flower graph $Z_v(h)$ with $h \geq 1$, odd harmonious labeling, and odd harmonious graphs. The data processing and analysis stage consists of constructing definitions and properties of new graphs. The result verification stage is in the form of making theorems about odd harmonious graphs with mathematical proof.

RESULTS AND DISCUSSION

The result of this research is constructing the definition of the zinnia flower graphs and their variations in Definition 1 and Definition 2.

Definition 1.

Zinnia flower graph $Z(h)$ with $h \geq 1$ is a graph with $V\big(Z(h)\big) = \{a_j \,|\, 1 \leq j \leq 2h+2\} \cup \{b_i | i = 1,2\} \cup \{c_j{}^i | 1 \leq j \leq h, i = 1,2\}$ and $E\big(Z(h)\big) = \{a_j b_i \,|\, 1 \leq j \leq 2h+2, i = 1,2\} \cup \{a_1 c_j{}^i \,|\, 1 \leq j \leq h, i = 1,2\} \cup \{a_2 c_j{}^i \,|\, 1 \leq j \leq h, i = 1,2\}$.

Such that $p = \big|V\big(Z(h)\big)\big| = 4h + 4$ and $q = \big|E\big(Z(h)\big)\big| = 8h + 4$ are obtained and the figure construction of the zinnia flower graph $Z(h)$ is obtained Figure 1.

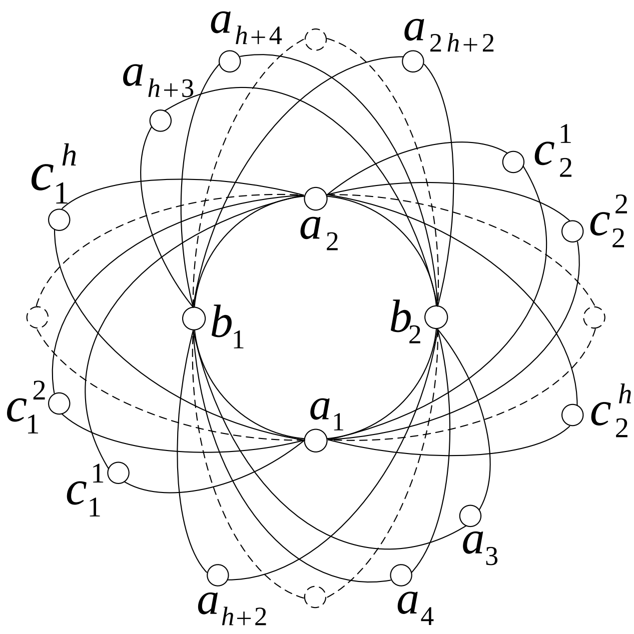

Figure 1. Zinnia flower graph $Z(h)$

Definition 2.

Variations of the zinnia flower graph $Z_v(h)$ with $h \geq 1$ is a graph with $V\big(Z_v(h)\big) = \{u_0\} \cup \{{v_j}^i \big| i,j = 1,2\} \cup \{w_j \big| 1 \leq j \leq h\} \cup \{x_j \big| 1 \leq j \leq h\} \cup \{y_j \big| 1 \leq j \leq h\} \cup \{z_j \big| 1 \leq j \leq h\}$ and $E\big(Z_v(h)\big) = \{u_0 {v_j}^i \big| \; i,j = 1,2\} \cup \{{v_1}^2 w_j \big| 1 \leq j \leq h\} \cup \{{v_2}^2 w_j \big| 1 \leq j \leq h\} \cup \{{v_2}^2 x_j \big| 1 \leq j \leq h\} \cup \{{v_2}^1 x_j \big| 1 \leq j \leq h\} \cup \{{v_2}^1 y_j \big| 1 \leq j \leq h\} \cup \{{v_1}^1 y_j \big| 1 \leq j \leq h\} \cup \{{v_1}^1 z_j \big| 1 \leq j \leq h\} \cup \{{v_1}^2 z_j \big| 1 \leq j \leq h\}$.

Such that $p = \big|V\big(Z_v(h)\big)\big| = 4h + 5$ and $q = \big|E\big(Z_v(h)\big)\big| = 8h + 4$ are obtained and the figure construction variations of the zinnia flower graph $Z_v(h)$ is obtained Figure 2.

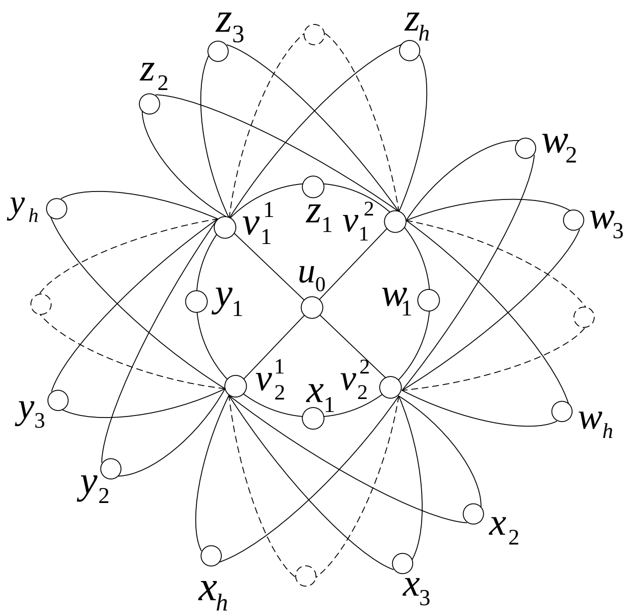

Figure 2. Variations of the zinnia flower graph $Z_v(h)$

Furthermore, it is proven that the zinnia flower graphs and their variations satisfy the properties of the odd harmonious labeling function stated in Theorem 3 and Theorem 4.

Theorem 3. Zinnia flower graph $Z(h)$ with $h \geq 1$ is an odd harmonious.

Proof.

Define the function $g: V(Z(h)) \to \{0,1,2,3 \ldots, 16h+7\}$ as follows

$$g(a_j) = 4j - 4, 1 \leq j \leq 2h+2 \quad (1)$$

$$g(b_i) = 2i - 1, i = 1,2 \quad (2)$$

$$g(c_j{}^i) = 8h + 8j + 2i - 1, 1 \leq j \leq h, i = 1,2 \quad (3)$$

Based on (1), (2) and (3), different labels are obtained and $V(Z(h)) \subseteq \{0,1,2,3 \ldots, 16h+7\}$, hence the vertex labeling function is injective.

Define the edge labeling function $g^*: E(Z(h)) \to \{1,3,5,7, \ldots, 16h+7\}$ as follows

$$g^*(a_j b_i) = 4j + 2i - 5, 1 \leq j \leq 2h+2, i = 1,2 \quad (4)$$

$$g^*(a_1 c_j{}^i) = 8h + 8j + 2i - 1, 1 \leq j \leq h, i = 1,2 \quad (5)$$

$$g^*(a_2 c_j{}^i) = 8h + 8j + 2i + 3, 1 \leq j \leq h, i = 1,2 \quad (6)$$

Based on (4), (5) and (6), different labels are obtained and $E(Z(h)) = \{1,3,5,7, \ldots, 16h+7\}$, hence the edge labeling function is bijective. Consequently zinnia flower graph $Z(h)$ is an odd harmonious. ■

Zinnia flower graph $Z(4)$ in Figure 3.

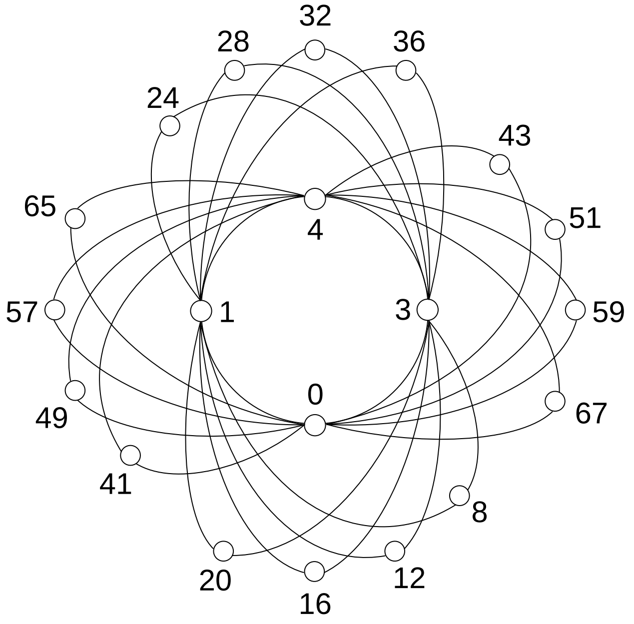

Figure 3. Zinnia flower graph $Z(4)$

Theorem 4. Variations of the zinnia flower graph $Z_v(h)$ with $h \geq 1$ is an odd harmonious.

Proof.

Define the function $g: V(Z_v(h)) \to \{0,1,2,3 \ldots, 16h+7\}$ as follows

$$g(u_0) = 0 \quad (7)$$

$$g(v_j{}^i) = 4j + 2i - 5, \; i, j = 1,2 \quad (8)$$

$$g(w_j) = 16j - 12, 1 \leq j \leq h \quad (9)$$

$$g(x_j) = 16j - 2, 1 \leq j \leq h \quad (10)$$

$$g(y_j) = 16j - 4, 1 \leq j \leq h \quad (11)$$

$g(z_j) = 16j + 2, 1 \leq j \leq h$ (12)

Based on (7), (8), (9), (10), (11) and (12), different labels are obtained and $V(Z_v(h)) \subseteq \{0,1,2,3 \ldots, 16h + 7\}$, hence the vertex labeling function is injective.

Define the edge labeling function $g^*: E(Z_v(h)) \to \{1,3,5,7, \ldots, 16h + 7\}$ as follows

$g^*(u_0 v_j{}^i) = 4j + 2i - 5,\ i, j = 1,2$ (13)

$g^*(v_1{}^2 w_j) = 16j - 7,\ 1 \leq j \leq h$ (14)

$g^*(v_2{}^2 w_j) = 16j - 5,\ 1 \leq j \leq h$ (15)

$g^*(v_2{}^2 x_j) = 16j + 5,\ 1 \leq j \leq h$ (16)

$g^*(v_2{}^1 x_j) = 16j + 1,\ 1 \leq j \leq h$ (17)

$g^*(v_2{}^1 y_j) = 16j - 1,\ 1 \leq j \leq h$ (18)

$g^*(v_1{}^1 y_j) = 16j - 3,\ 1 \leq j \leq h$ (19)

$g^*(v_1{}^1 z_j) = 16j + 3,\ 1 \leq j \leq h$ (20)

$g^*(v_1{}^1 z_j) = 16j + 7,\ 1 \leq j \leq h$ (21)

Based on (13), (14), (15), (16), (17), (18), (19), (20) and (21), different labels are obtained and $E(Z_v(h)) = \{1,3,5,7, \ldots, 16h + 7\}$, hence the edge labeling function is bijective. Consequently variations of the zinnia flower graph $Z_v(h)$ is an odd harmonious. ■

Variations of the zinnia flower graph $Z(5)$ in Figure 4.

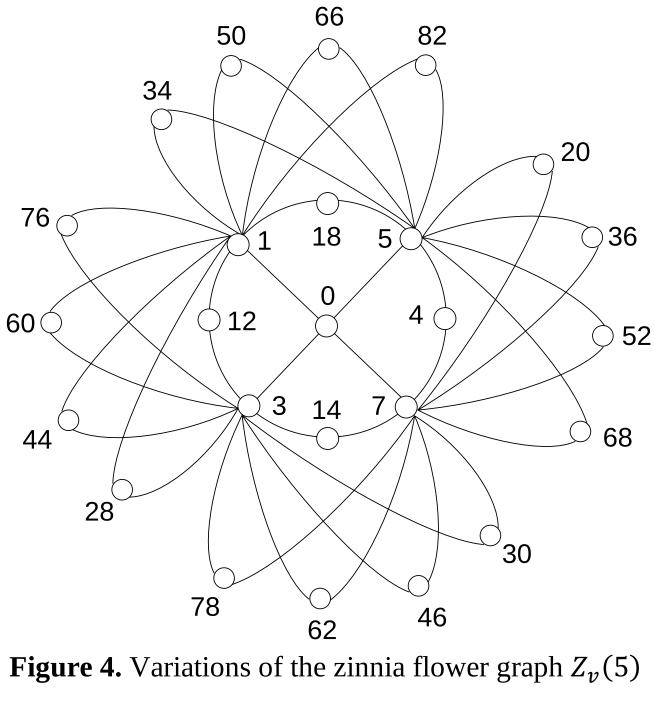

Figure 4. Variations of the zinnia flower graph $Z_v(5)$

Based on Theorem 3 and Theorem 4, it has been obtained that $Z(h)$ with $h \geq 1$ in Definition 1 and variations of the zinnia flower graph $Z_v(h)$ with $h \geq 1$ are odd harmonious labeling.

CONCLUSION

The conclusion of this research is the definition of the zinnia flower graph $Z(h)$ with $h \geq 1$ in Definition 1 and variations of the zinnia flower graph $Z_v(h)$ with $h \geq 1$ in Definition 2.. Theorem 3 for the proof of the zinnia flower graph and Theorem 4 for the proof of the zinnia flower graph variation. The benefit of this result is that a new graph classes zinnia flower graph $Z(h)$ with $h \geq 1$ in Definition 1 and variations of the zinnia flower graph $Z_v(h)$ with $h \geq 1$ has been discovered is an odd harmonious graphs. For further research, this study can be continued by looking for the construction of new graph class definitions that are the development of the zinnia flower graph and proving that these graphs are also odd harmonious graphs.

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