e-ISSN 2540-9840 p-ISSN 1412-3770 Jurnal Ilmiah Sains, April 2024, 24(1): 28-37
Accredited by Ministry of Education, Culture, Research and Technology No: 105/E/KPT/2022
DOI: https://doi.org/10.35799/jis.v24i1.51387
Available online at https://ejournal.unsrat.ac.id/index.php/jis

Susceptible-Infected-Recovered Model of Mathematics Anxiety Behavior on Students' Mathematics Study Results at Aquino Catholic High School Amurang

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ABSTRACT

This study aims to see the spread of mathematics anxiety behavior on the results of students' mathematics studies at Aquino Catholic High School Amurang by using the SIR (Susceptible-Infected-Recovered) mathematical model. A total of 88 samples were collected and categorized into three groups based on predetermined criteria. These groups consisted of 47 students categorized as susceptible, 27 as infected, and 14 as recovered. The parameters in this study measure the rate of change within these three groups over a 365-day period, equivalent to one year. Two equilibrium points are obtained that can interpret populations free from math anxiety behavior or endemic math anxiety behavior. The stability analysis of the two equilibrium points shows that the equilibrium point free from math anxiety behavior is locally asymptotically stable. Additionally, this study also reveals that math anxiety behavior will disappear in less than 40 days through the basic reproduction number (R_0) obtained, which is 0.99 or less than 1.

Keywords: Amurang; mathematics anxiety; SIR model

Model SIR (Susceptible-Infected-Recovered) terhadap Perilaku Kecemasan Matematika (Mathematics Anxiety) pada Hasil Studi Matematika Siswa di SMAS Katolik Aquino Amurang

ABSTRAK

Penelitian ini bertujuan untuk melihat penyebaran perilaku kecemasan matematika (mathematics anxiety) pada hasil studi matematika siswa di SMAS Katolik Aquino Amurang dengan menggunakan model matematika SIR (Susceptible-Infected-Recovered). Sebanyak 88 sampel telah diambil untuk dibagikan ke dalam tiga kelas berdasarkan asumsi-asumsi yang telah dibangun, yaitu kelas susceptible (rentan) sebanyak 47 siswa, infected (terinfeksi) sebanyak 27 siswa, dan recovered (pulih) sebanyak 14 siswa. Parameter dalam penelitian ini menunjukkan laju perubahan nilai ketiga kelas tersebut dalam kurun waktu 365 hari atau 1 tahun. Diperoleh dua titik kesetimbangan yang dapat menginterpretasikan populasi bebas perilaku kecemasan matematika ataupun endemik perilaku kecemasan matematika. Analisis kestabilan dua titik kesetimbangan itu menunjukkan bahwa titik kesetimbangan bebas perilaku kecemasan matematika bersifat stabil asimtotik lokal. Selain itu penelitian ini juga menunjukkan perilaku kecemasan matematika akan menghilang dalam kurun waktu kurang dari 40 hari lewat bilangan reproduksi dasar (R_0) yang diperoleh yaitu sebesar 0,99 atau $<$ 1.

Kata kunci: Amurang; Kecemasan matematika; model SIR

(Article History: Received 30-09-2023; Accepted 10-04-2024; Published 11-04-2024)

INTRODUCTION

Anxiety disorders are among the most prevalent mental health conditions worldwide (Luttenberger *et al*., 2018). (Marsh & Shanks, 2014) Conducting an experiment on the belief that mental illnesses like anxiety, severe depression, and others can be contagious and contribute to social behaviors. Many participants in the experiment agreed that mental disorders can be transmitted from one person to another, and individual perspectives on the transmissibility of such disorders can influence their willingness to interact with those who have them. In educational settings, anxiety can significantly impede learners, particularly when it pertains to tasks requiring greater cognitive effort. Notably, one significant example of this is mathematics anxiety (Blazer, 2011).

High school students in Indonesia typically fall within the age range of 15-18 years. According to the World Health Organization (WHO, 2018), high school students can be categorized as adolescents, as adolescence spans from ages 10 to 19. During adolescence, individuals often experience internal and external changes that can lead to inner turmoil, rendering them susceptible to various influences. These changes, whether internal or external, can precipitate the emergence of anxiety.

Mathematics anxiety can be defined as the experience of fear and heightened physiological reactivity when individuals confront mathematical tasks, such as manipulating numbers, solving mathematical problems, or facing evaluative situations related to mathematics (Paechter *et al*., 2017). This anxiety toward mathematics manifests in three key aspects: emotional, cognitive, and physiological, each of which can detrimentally impact an individual's performance. Emotionally, individuals may experience worry, tension, anxiety, and restlessness (Tuma & Maser, 2019). Cognitively, mathematics anxiety affects an individual's working memory function (Luttenberger *et al.*, 2018). Meanwhile, physiologically, mathematics anxiety can result in various symptoms, including an elevated heart rate, sweaty palms, stomachaches, and dizziness (Blazer, 2011). A neurocognitive study has shown that responses and effects related to math anxiety are associated with fear and pain networks in the brain (Artemenko *et al*., 2015). In a functional MRI study, pain network activity in the insula can be observed when individuals experience anxiety regarding mathematical tasks (Lyons & Beilock, 2012). Meanwhile, research focused on the amygdala network indicates that children with high math anxiety exhibit hyperactivity and abnormal connectivity in the right basolateral amygdala, suggesting that the brain's response to math anxiety in this network is age-dependent (Artemenko *et al.*, 2015).

One valuable tool for addressing real-world problem-solving is mathematical modeling, wherein these real-world issues are translated into mathematical forms using specific assumptions. Among the myriad real-world issues, disease spread is one problem amenable to analysis using the SIR model (Susceptible-Infected-Recovered). The SIR model categorizes the population into three conditions: susceptible (S), where individuals are healthy but can contract the disease; infected (I), where individuals are infected and can transmit it; and recovered (R), where individuals have recovered and are immune to the disease (Sari & Tasman, 2014; Chamnan & Pongsumpun, 2021). In a closed population, the spread of contagious diseases leads to changes in the numbers of each group in each part of the population (Yudasubrata, 2018; Mangobi *et al*., 2023)).

Numerous prior studies have explored the use of the SIR model and examined the influence of mathematics anxiety on students' academic performance. For instance, (Agus *et al.*, 2020) employed the SIR epidemiological model to analyze mobile game usage among students in the Faculty of Mathematics and Natural Sciences at the University of West Sulawesi. Additionally, (Side *et al.*, 2020) applied the mathematical SIR model to address issues related to social media addiction. Furthermore, (Qausarina, 2016) scrutinized the impact of math anxiety on students' math learning outcomes, focusing on 10th-grade students at SMA Negeri 11 Banda Aceh, using regression and correlation analyses. Building on these prior studies, this research underscores the importance of conducting research using the SIR model to address mathematics anxiety issues and ascertain whether mathematics anxiety behavior dissipates or persists within a population.

This research aims to determine two equilibrium points signifying a population free of mathematics anxiety behavior and a population endemic to mathematics anxiety behavior. Additionally, it seeks to perform stability analysis of these equilibrium points using the Jacobian matrix to identify eigenvalues (λ) and discern whether mathematics anxiety behavior will dissipate, remain stable, or increase within a specific timeframe within the population.

RESEARCH METHOD

This research was conducted between March and August 2023. Data processing and analysis were performed at the Computer Laboratory of the Department of Mathematics, Faculty of Mathematics and Natural Sciences, Sam Ratulangi University, Manado.

The data utilized in this study were primary data collected through the distribution of questionnaires to 88 students in the 11th grade during the academic year 2022/2023 at Aquino Catholic High School Amurang. The students' mathematics performance data were derived from their raw scores in the final semester mathematics exams, encompassing students from the 11th grade across the Science, Social Sciences, and Humanities streams.

Closed questionnaires were employed, enabling students to choose responses from provided options. The mathematics anxiety questionnaire for students employed a Likert scale with a range of agreement levels from 1 to 5 choices, spanning from 'Strongly Agree' to 'Strongly Disagree'.

A Likert scale calculation guide was utilized to classify students into two categories based on their levels of mathematics anxiety. Students with low levels of math anxiety were categorized with questionnaire scores in the range ≥ 14 and < 42, while those with high levels of math anxiety fell within the range ≥ 42 and ≤ 70.

The variables used in this research are described in Table 1.

Table 1. Research Variables

Variable	Description
N	Total student sample
S	The number of students vulnerable to experiencing math anxiety based on student questionnaire scores in the low category.
I	The number of students experiencing math anxiety based on student questionnaire scores in the high category.
R	The number of students who have met the minimum passing criteria, which is a score of 75 out of 100 on the final semester mathematics exam for students.
A	The rate of students currently studying mathematics.
β	The rate of students experiencing mathematics anxiety.
α	The rate of students with questionnaire scores in the high category who still meet the minimum passing criteria, which is a score of 75 out of 100 on the final semester mathematics exam for students.
ε	The rate of students with questionnaire scores in the low category who still meet the minimum passing criteria, which is a score of 75 out of 100 on the final semester mathematics exam for students.
μ	Natural deaths based on the life expectancy at Aquino Catholic High School Amurang.

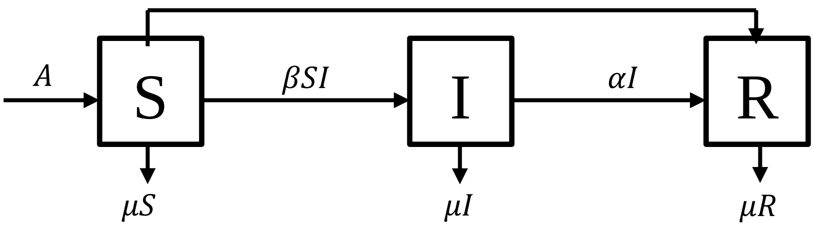

Figure 1. Flow of Mathematics Anxiety Behavior Spread

From the model in Figure 1, we obtain the system of differential equations as follows:

$$\frac{dS}{dt} = A - \beta SI - (\varepsilon + \mu)S$$
$$\frac{dI}{dt} = \beta SI - (\alpha + \mu)I \qquad (1)$$
$$\frac{dR}{dt} = \varepsilon S + \alpha I - \mu R$$

with $S + I + R = N$ representing the total population.

Equilibrium Points of Mathematics Anxiety Behavior

1. Equilibrium point free of mathematics anxiety behavior $TE_0 = \left\{\frac{A}{\varepsilon+\mu}, 0, \frac{A\varepsilon}{(\varepsilon+\mu)\mu}\right\}$.
2. Endemic mathematics anxiety behavior equilibrium point $TE_1 = \left\{\frac{\alpha+\mu}{\beta}, \frac{A\beta-\alpha\varepsilon-\varepsilon\mu-\alpha\mu-\mu^2}{\beta(\alpha+\mu)}, \frac{A\alpha\beta+\alpha\varepsilon\mu-\alpha\mu^2+\varepsilon\mu^2-\alpha^2\mu}{\beta(\alpha+\mu)\mu}\right\}$.

Data Simulation

The classification of Susceptible (S) and Infected (I) classes is determined based on students' questionnaire scores, where the Susceptible (S) class includes students with low-level mathematics anxiety and the Infected (I) class includes students with high-level mathematics anxiety criteria. The variables used in the analysis of this research model are presented in the table below:

Table 2. Initial Data of Students Experiencing Mathematics Anxiety

Variable	Value	Unit
S	60	Person
I	28	Person
N	88	Person

The population of students who have recovered is obtained from the population of students who were susceptible to experiencing math anxiety but met the minimum passing grade criteria on their end-of-semester exam scores. This also includes the population of students who experienced math anxiety but met the minimum passing grade criteria on their end-of-semester exam scores. Thus, the variables used in the analysis of this research model are as follows.

Table 3. Data of Students Experiencing Mathematics Anxiety

Variable	Value	Unit
S	47	Person
I	27	Person
R	14	Person
N	88	Person

The natural mortality rate is obtained based on the life expectancy at Catholic Aquino High School in Amurang, which is 3 years. Based on the data in Table 3 and the student math anxiety questionnaire data, the values of the parameters to be used in simulating the data are as follows.

$$A(t) = \frac{S}{N} = \frac{47}{88} = 0.534090909$$

$$\varepsilon(t) = \frac{Revovery\ from\ S}{N} = \frac{13}{88} = 0.147727273$$

$$\beta(t) = \frac{I}{N} = \frac{27}{88} = 0.306818182$$

$$\alpha(t) = \frac{Recovery\ from\ I}{N} = \frac{1}{88} = 0.0113636364$$

$$\mu(t) = \frac{1}{Natural\ mortality\ rate} = \frac{1}{3} = 0.333333333$$

Table 4. Parameters of SIR Model of Mathematics Anxiety Behavior

Parameter	Value	Unit
A	0.534090909	Person/day
ε	0.147727273	Person/day
β	0.306818182	Person/day
α	0.0113636364	Person/day
μ	0.333333333	Person/day

Stability of the Free Equilibrium Point in Mathematics Anxiety Behavior

Jacobian matrix of the free equilibrium point for mathematics anxiety behavior.

$$Jf(TE_0) = \begin{bmatrix} -\varepsilon-\mu & \frac{-A\beta}{\varepsilon+\mu} & 0 \\ 0 & \frac{A\beta}{\varepsilon+\mu}-\alpha-\mu & 0 \\ \varepsilon & \alpha & -\mu \end{bmatrix}$$

the eigenvalues obtained from the determinant of the matrix $Jf(TE_0)$ are as follows.

1) $\lambda_1 = -\varepsilon - \mu$ (2)
2) $\lambda_2 = \frac{A\beta}{\varepsilon+\mu} - \alpha - \mu$ (3)
3) $\lambda_3 = -\mu$ (4)

Then substitute the parameter values in table 4 into each eigenvalue above, thus obtaining $\lambda_1 = -0.481060606, \lambda_2 = -0.004056311$, $\lambda_3 = -0,333333333$. Because all three eigenvalues are negative or less than zero, according to the Olsder and Woude theorem, the free equilibrium point of mathematics anxiety behavior is locally asymptotically stable. This can be observed in Figure 2.

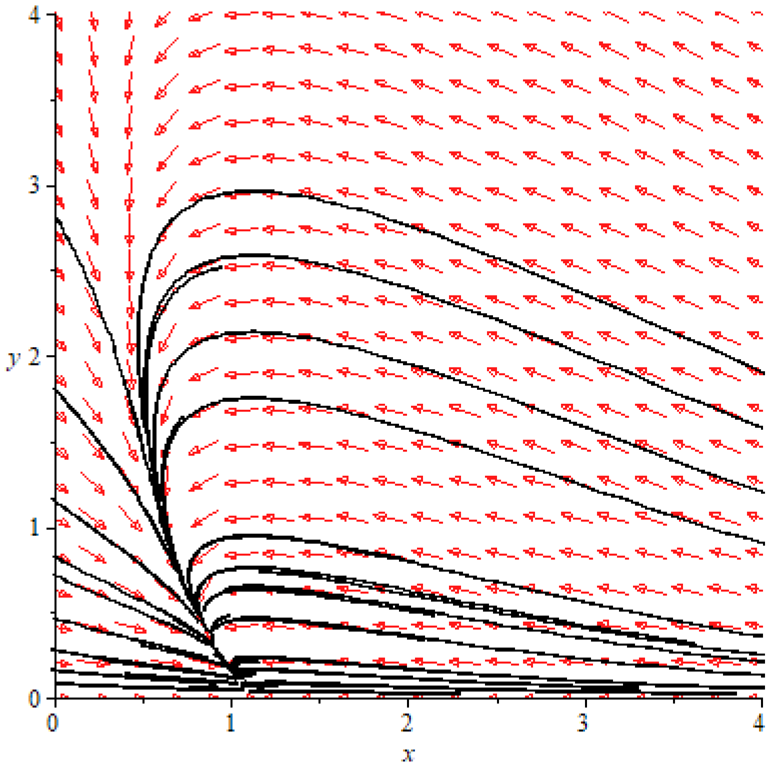

Figure 2. The Stability Orbit of the Free Equilibrium Point of Mathematics Anxiety Behavior

It can be seen that the trajectory is directed towards the equilibrium point, thus it can be concluded that the free equilibrium point of math anxiety behavior is locally asymptotically stable.

Stability of the Endemic Equilibrium Point for Mathematics Anxiety

Jacobian matrix of the endemic equilibrium point for mathematics anxiety behavior.

$$Jf(TE_1) = \begin{bmatrix} \frac{-A\beta - \alpha\varepsilon - \varepsilon\mu - \alpha\mu - \mu^2}{\alpha + \mu} - \varepsilon - \mu & -\alpha - \mu & 0 \\ \frac{A\beta - \alpha\varepsilon - \varepsilon\mu - \alpha\mu - \mu^2}{\alpha + \mu} & 0 & 0 \\ \varepsilon & \alpha & -\mu \end{bmatrix}$$

the eigenvalues obtained from the determinant of the matrix $Jf(TE_1)$ are as follows.

1) $\lambda_1 = -\mu$ (5)

2) $$\lambda_2 = \frac{\frac{-A\beta-\alpha\varepsilon-\varepsilon\mu-\alpha\mu-\mu^2}{\alpha+\mu} - \varepsilon - \mu + \sqrt{\left(\frac{-A\beta-\alpha\varepsilon-\varepsilon\mu-\alpha\mu-\mu^2}{\alpha+\mu} - \varepsilon - \mu\right)^2 + 4(\alpha+\mu)\left(\frac{-A\beta+\alpha\varepsilon+\varepsilon\mu+\alpha\mu+\mu^2}{\alpha+\mu}\right)}}{2} \quad (6)$$

3) $$\lambda_3 = \frac{\frac{-A\beta-\alpha\varepsilon-\varepsilon\mu-\alpha\mu-\mu^2}{\alpha+\mu} - \varepsilon - \mu - \sqrt{\left(\frac{-A\beta-\alpha\varepsilon-\varepsilon\mu-\alpha\mu-\mu^2}{\alpha+\mu} - \varepsilon - \mu\right)^2 + 4(\alpha+\mu)\left(\frac{-A\beta+\alpha\varepsilon+\varepsilon\mu+\alpha\mu+\mu^2}{\alpha+\mu}\right)}}{2} \quad (7)$$

Then substitute the parameter values in table 4 into each eigenvalue above, thus obtaining $\lambda_1 = -0{,}333333333$, $\lambda_2 = 0{,}001356149$, $\lambda_3 = -1{,}438876962$. Because one of the eigenvalues is positive or greater than zero, according to the Olsder and Woude theorem, the endemic equilibrium point of mathematics anxiety behavior is unstable. This can be observed in Figure 3.

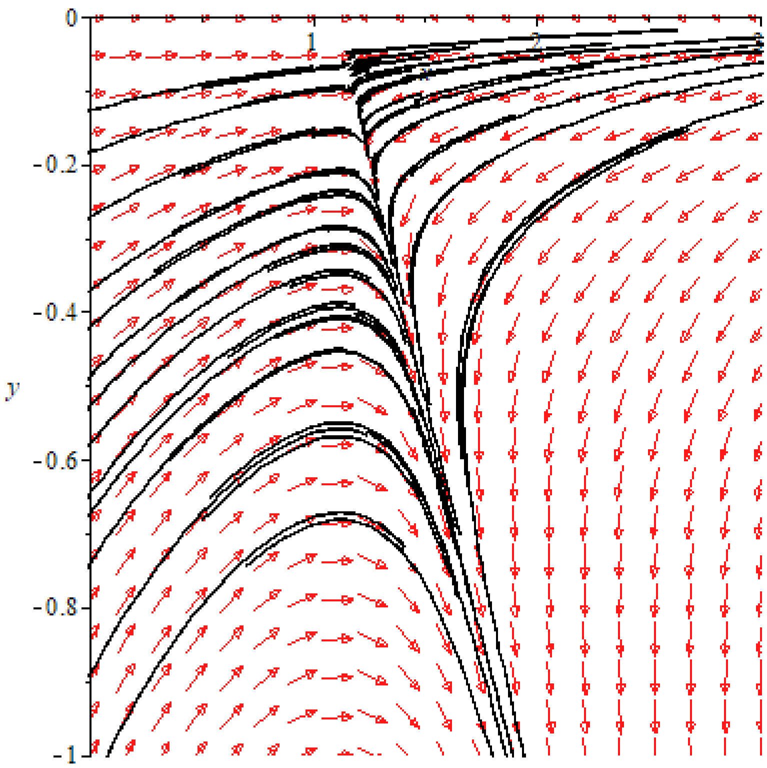

Figure 3. The Stability Orbit of the Endemic Equilibrium Point for Mathematics Anxiety Behavior

it can be seen that the trajectory is moving away from the equilibrium point, thus it can be concluded that the endemic equilibrium point of math anxiety behavior is unstable.

Basic Reproduction Number

Basic reproduction number can indicate the number of individuals in a class susceptible to experiencing math anxiety caused by one individual experiencing math anxiety. The greater the value of the basic reproduction number, the more it indicates that math anxiety is more easily spread and vice versa.

The Basic Reproduction Number (R_0) of the SIR model for student mathematics anxiety behavior in the context of mathematics study results is as follows.

$$R_0 = \frac{A\beta}{\alpha\varepsilon+\alpha\mu+\varepsilon\mu+\mu^2} \quad (8)$$

Then substitute the parameter values from Table 4 into equation (8) to obtain the value of the Basic Reproduction Number (R_0).

$R_0 = 0{,}988232242$

this means that, on average, one student experiencing math anxiety can transmit their math anxiety to approximately 0.99 students. The calculated basic reproduction number (R_0) from this data simulation is less than one. Therefore, math anxiety behavior is expected to decline in the population, and it can be interpreted that math anxiety will not become an epidemic.

Simulation of the SIR Model for Mathematics Anxiety Behavior

Simulation of the SIR model for math anxiety behavior based on the math study results of students at Catholic Aquino High School in Amurang can be illustrated as follows.

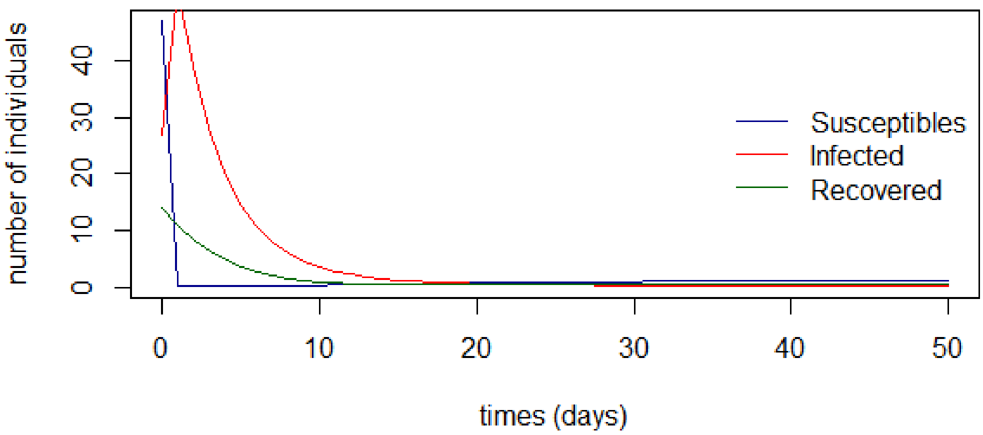

Figure 4. Plot of the SIR Model for Math Anxiety Behavior Against Students' Mathematics Study Results.

The time period used in this data plot is 50 days. It can be observed that in the susceptible class, there is a significant reduction in the number of students in less than 10 days, while at the same time, the infected class experiences a surge from 27 students to over 50 students, or approximately 190% of its initial value.

It takes less than 40 days for the susceptible (S), infected (I), and recovered (R) classes to have nearly the same values, approaching zero and reaching a stagnant or stable state. Therefore, it can be concluded that math anxiety behavior with respect to students' math study results will not become endemic within the student population at Catholic Aquino High School in Amurang.

CONCLUSION

There are two equilibrium points that represent the population in this research: the free equilibrium point for mathematics anxiety behavior $TE_0 = \left\{\frac{A}{\varepsilon+\mu}, 0, \frac{A\varepsilon}{(\varepsilon+\mu)\mu}\right\}$, and the endemic equilibrium point for mathematics anxiety behavior $TE_1 = \left\{\frac{\alpha+\mu}{\beta}, \frac{A\beta-\alpha\varepsilon-\varepsilon\mu-\alpha\mu-\mu^2}{\beta(\alpha+\mu)}, \frac{A\alpha\beta+\alpha\varepsilon\mu-\alpha\mu^2+\varepsilon\mu^2-\alpha^2\mu}{\beta(\alpha+\mu)\mu}\right\}$. Stability analysis of both equilibrium points shows that the free equilibrium point for mathematics anxiety behavior is locally asymptotically stable, while the endemic equilibrium point for mathematics anxiety behavior is unstable.

Basic reproduction number (R_0) obtained from the data simulation is $0{,}988232242$, this value indicates that math anxiety behavior will become extinct in the population. There is a relationship between the results of the stability analysis of both equilibrium points and the basic reproduction number. The two resulting values indicate that the mathematics anxiety behavior will not become endemic in the population of Aquino Catholic High School Amurang. In this case, it only takes less than 40 days for mathematics anxiety to disappear within the population.

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