**ENDOMORPHISM REPRESENTATION MATRIX FROM STANDARD GENETIC CODE**

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**ABSTRACT**

Mutations are changes in genetic material that can occur at the level of genes or chromosomes. Mutations at the gene level are structural changes in the genetic code. Mathematically, genetic mutations can be viewed with an endomorphism and automorphism f in the vector space which maps the standard genetic code sequence of wild-type genes into mutant genes. In the endomorphism there is a matrix called the endomorphism representation matrix. An endomorphism f is called an automorphism at if and only if each diagonal element of the endomorphism representation matrix is ​​an odd number.

Keywords: authomorphism, endomophism, mutation

**INTRODUCTION**

Genetic code can be viewed as a set of rules that define how a nucleotide sequence in a DNA or RNA determines a particular amino acid sequence in protein synthesis .In the Standard genetic code, there is often a structural change at the level of genes known as mutations.which a change in the DNA sequence of a living thing. Mutations can occur due to chemical effects, physical effects or there is an error when replicating DNA [1].

Mutations are also one of the factors that determine evolution in living things. Mutations that run continuously can lead to the emergence of new varieties that are different from their ancestors which resulted in the evolutionary process. Evolution in the study of biology means a change in the inherited characteristics of a population of organisms from one to the next generations.

When organisms reproduce, their off spring will bring new traits. This new trait can be obtained from genetic changes due to genetic mutations or gene transfer between populations and between species. Mutations that cause evolution in living things that reproduce will only occur if these mutations occur in gamete cells.

It has been many studies that represent genetic mutations algebra, including Sanchez et al., In 2004 by the journal entitled "*The Genetic Code Boolean Lattice*", Jose et al., In 2007 in jurnah entitled "*An Extended RNA code and its Relationship to The Standard Genetic Code: An Algebraic and Geometrical Approach ".*

The set of nucleotide bases can be matched with a member of the set of integers ,, and can be viewed as a set of structures that form a group with a binary operation. Then set the standard genetic code expanded into a set of rows of standard genetic code as a result of a matching set of bases with integers can be reviewed algebraic structure. The set of the standard genetic code line in genetics is a member of the DNA.

Genetic mutations can be reviewed in algebra, by satisfying certain properties. Sanchez, R. et al., In 2005 in the journal entitled "Gene Algebra from a Genetic Structure Algebraic Code" suggests that the genes, a similar situation arises in which almost all mutations in codon associated with automorfisma. On the basis of these problems, in this study the authors will represent a genetic mutation using automorfisma on space-algebra

R-Algebraic of Standard Genetic Code Sets

 Nucleotide bases can be seen as an ordered set obtained by adjusting the difference in the number of hydrogen bonds between bases and the chemical properties (purines and pyrimidines) of bases. Nucleotide bases that have 2 hydrogen bonds are adenine (A) and thymine (T) / uracil (U), while nucleotide bases that have 3 hydrogen bonds, namely guanine (G) and cytosine (C).[8]

This sequence arrangement starts by selecting the nucleotide base that has the least hydrogen bonds so that you can choose A or U. If U is chosen, then G will then be chosen because it has different chemical properties than U. Furthermore, C is chosen because of the different chemical properties from G, and finally A is chosen because of differences in chemical properties with C, so that the obtained set {U, G, C, A} is obtained. If the first base chosen is A, then with the same sorting rule we get an ordered set {A, C, G, U} [9,10].

The set of nucleotide bases is converted to a set of standard genetic code (triplet nucleotide) denoted by Conversion is done by sorting three nucleotide bases as genetic code so that there is a 4 × 4 × 4 = 64 possible sequence formed as a standard genetic codes. Each member of will be matched with a member set of .

The standard genetic code set be expanded to the standard genetic code set using the direct sum of N times denoted by so the number of members of the set is . can be viewed as a commutative ring and left module over ring , so is algebra [9,10]

Algebra structure of standard genetic Code

is vector spaces over . The basis for this vector space is the set of the following vectors

so that each DNA sequence α ∈ over the ring has their respective representations as where (), in other words ordered pairs is the coordinate representation of the gene α∈ in .

Each element can be represented by ring , because is -algebra [11], and (.

Each element of the group can be grouped into seven classes. According to the Lagrange theorem [2,3,4,5], the order of the is a divisor of 64 and the order has the form . Elements that have the order are the form , where x is an odd integer between 1 and ( -1).

The grouping of elements from groups (, +) can be seen from the following table 1:

T**able 1** Partitions from Groups (, +) into Seven Classes according to the order

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Order | 1 | 2 | 4 | 8 | 16 | 32 | 64 |
| M | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| Elemen | 0 | 32 | 16 | 8 | 4 | 2 | 1 | 1 |
|  |  | 48 | 24 | 12 | 6 | 3 | 43 |
|  |  |  | 40 | 20 | 10 | 5 | 13 |
|  |  |  | 56 | 28 | 14 | 7 | 55 |
|  |  |  |  | 36 | 18 | 9 | 57 |
|  |  |  |  | 44 | 22 | 11 | 35 |
|  |  |  |  | 52 | 26 | 15 | 47 |
|  |  |  |  | 60 | 30 | 17 | 49 |
|  |  |  |  |  | 34 | 19 | 27 |
|  |  |  |  |  | 38 | 21 | 61 |
|  |  |  |  |  | 42 | 23 | 39 |
|  |  |  |  |  | 46 | 25 | 41 |
|  |  |  |  |  | 50 | 29 | 53 |
|  |  |  |  |  | 54 | 31 | 31 |
|  |  |  |  |  | 58 | 33 | 33 |
|  |  |  |  |  | 62 | 37 | 45 |
|  |  |  |  |  |  | 51 | 59 |
|  |  |  |  |  |  | 63 | 63 |

Elements with order 64 are loaded into two columns, where each same row is an inverse of the multiplication operation for the ring.

Endomorphism of genetic mutations

 Gene mutations can be seen as linear transformations f (α) = β which is endomorphism in vector space with α ∈ is a wild-type gene and β ∈ is a mutant gene of α.

Genetic mutations can be seen as endomorphism f at . Mapping construction f: → , which maps the standard genetic code sequence of wild-type genes into the standard genetic code sequence of mutant genes.

An endomorphism f: → can be called a local endomorphism if there are and with such that:, for and , so for each

 , .

Local endomorphism f is called diagonal if and  for , in other words .

If the representation matrix endomorphism f which depends on the canonical base is a diagonal matrix, then it is called the diagonal endomorphism matrix f.

There is a relationship between algebraic properties and biological properties in above algebra -algebra in terms of the order of each element [10]. Quantitatively explained in the following theorem.

Theorem 1 :

For each α, β∈ there is endomorphism f: → , so f (α) = β if and only if order β divides the order α.

Proof:

1.It is known that is an isomorphic abelian group with abelian groups and f: → , endomorphism so f (α) = β. We will proof order β divides the order α.

Because *f* is a mapping that is surjectif, then every element in the range has a pre-map, so the range is a subgroup ,based on the Lagrange theorem, so β divides α .

2.Next, consider , is endomorphism and for each α, β∈ , order β divides order α, will prove to be f (α) = β. Let α, β ∈ in the order (α) = and order (β) = with and with is the element with the maximum order at .

Based on the definition of basis, has a basis where and have a base where (). Because there is uniqu automorphismsuch that for , then endomorphism f mapping α to β, thus if order divide α, then f (α) = β.

From this theorem, this corollary is obatained.

Corollary 1

There is a diagonal endomorphism which converts vectors α to vectors β ,where α, β∈ if and only if for each satisfies

**Representatin Matrix of genetic mutation**

Diagonal endomorphism *f* is said to be automorphism at if and only if the is an odd number [11].

For each endomorphism or automorphism , there is a matrix A called the representation matrix of endomorphism f which depends on the canonical basis ordo N × N:

where the row entries are vector images

Based on corrolary 1, if and with is an odd number, the diagonal from matrix A can be determined by the equation :

Automorphism can be used to study gene mutations in N-dimensional space[9,10]. Therefore a matrix of representation of any automorphism between two genes will be determined. An endomorphism is an automorphism if and only if the diagonal element is odd.

Suppose A is a matrix of representation of local endomorphism , where with and . Let with vector elements is a linear combination of column elements to -l from endomorphism matrix A, in other words , so

|  |  |
| --- | --- |
|  |  |

 (2)

Next, to change the diagonal element of the endomorphism f representation matrix, the following theorem is used:

Theoreme 2:

Suppose is order of and is order . . If the elements are pivots and for k ≠ i then .

Proof

Suppose the matrix A is a representation matrix of local endomorphism f such that :

 and where and are odd numbers, then equation causes Because in the abelian group for each ,, the equation ,, has a solution if

Specifically, according to equation (2), the element can be taken as a pivot and the element for k ≠ i can be chosen arbitrarily for . So, we can specify the column element to l.

Conclusion

1. There is a mapping that maps the standard genetic code sequence of wild-type genes into the standard genetic code sequence of mutant genes, namely endomorphism .

2. Representation of genetic mutations can be done by forming an automorphism representation matrix. The diagonal endomorphism representation matrix f is said to be automorphism at .if and only if the element is an odd number.

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