Region Coloring in Minahasa Regency Using Sequential Color Algorithm

Yevie Ingamita[1], Nelson Nainggolan [1], Benny Pinontoan[1*]

[1]Mathematics Department–Faculty of Mathematics and Natural Sciences– University of Sam Ratulangi Manado, Indonesia

[*]Corressponding author : bpinto@yahoo.com

A B S T R A C T

Graph Theory is one of the mathematical sciences whose application is very wide in human life. One of theory graph applications is Map Coloring. This research discusses how to color the map of Minahasa Regency by using the possible minimum color. The algorithm used to determine the minimum color in coloring the region of Minahasa Regency is Sequential Color Algorithm. The Sequential Color Algorithm is an algorithm used in coloring a graph with k-color, where k is a possitive integer. Based on the results of this research, the Sequential Color Agorithm can be used to color the map of Minahasa Regency with the minimum number of colors or chromatic number $\chi(G)$ obtained in the coloring of 25 sub-districts on the map of Minahasa Regency are 3 colors ($\chi(G) = 3$).

A B S T R A K

Teori graf merupakan salah satu ilmu matematika yang penerapannya sangat luas dalam kehidupan manusia. Salah satu penerapan ilmu teori graf adalah pewarnaan peta. Penelitian ini membahas tentang bagaimana pewarnaan peta wilayah Kabupaten Minahasa dengan menggunakan jumlah warna yang sesedikit mungkin. Algoritma yang digunakan untuk menentukan warna paling sedikit dalam mewarnai wilayah Kabupaten Minahasa yaitu Algoritma *Sequential Color*. Algoritma *Sequential Color* adalah algoritma yang digunakan dalam mewarnai sebuah graf degan k-warna, dengan k adalah bilangan integer positif. Berdasarkan hasil penelitian ini didapatkan bahwa Algoritma *Sequential Color* dapat digunakan untuk melakukan pewarnaan peta wilayah Kabupaten Minahasa dengan jumlah warna minimum atau bilangan kromatik $\chi(G)$ yang didapatkan pada pewarnaan 25 kecamatan di Peta Kabupaten Minahasa adalah 3 warna ($\chi(G) = 3$).

1. INTRODUCTION

Graph theory is a very interesting subject to be explored because it has a variety of models whose application is very broad on solving problems of human life today. Although graph theory is part of exact sciences, i.e. mathematics, which in ordinary societies is only known by general formulas for solving real mathematical problems, in actual application models of graph theory can be associated with other fields of science such as communication networks, chemistry, biology, computer sciences, transportation, operations research, and so on.

Graph theory has existed since 1736 through Euler's book about problem-solving efforts on the Konigsberg bridge which was one of the famous bridges in Europe at that time. In daily life, the graph is useful for describing various structures that represent several objects so they are easy to understand. For example, on a map, cities are represented as vertices and roads that connect the cities are represented as edges.

One of the graph theory's models that can be applied in daily life is map coloring. The graph here is represented as a map. The coloring is done in 3 ways that are by coloring the vertices, coloring the edges, then coloring the regions. The aim is to find out the least number of colors from a map called chromatic numbers.

Nowadays, there are so many algorithms that can be used for graph coloring. The sequential color algorithm is one of them. The regional coloring of a graph in its application is to color the area on the map, provided that no area of the same color is neighboring. The colors used also can be minimized by this algorithm.

The coloring of the map that uses minimum colors will facilitate coloring the map with a very large area. The map does not have to be colored with the same number of colors as the number of areas that will be colored but only by using the minimum color, the areas on the map can be sorted between one area and another.

The sequential color algorithm is to coloring a graph with color, which is a positive integer. The

method used is coloring graph directly with minimum colors [1].

Referring to the previous research [2] that applied on the map in Kuantan Singingi Regency, coloring on maps can be modeled using graph theory. The use of graph theory is specialized in map coloring and the number of colors that used to color the area on the map. Objects represented as dots or vertices, while the regional boundaries represented as lines or edges. The solution is done by vertices coloring method using the Sequential Color Algorithm.

Based on the background described above, the author wants to do a map coloring research using Sequential Color Algorithm, so the title that the author appoints is "Region Coloring in Minahasa Regency using The Sequential Color Algorithm".

History of Graph Theory

The origin of graph theory started with the problem of Konigsberg bridge, in 1735. This problem lead to the concept of Eulerian Graph. Euler studied the problem of Konigsberg bridge and constructed a structure to solve the problem called Eulerian graph [3].

A graph is used to represent discrete objects and relationships between these objects. The visual representation of a graph is to declare an object expressed with a dot, circle, or point, while the relationship between objects is expressed as a line or edges [4].

Graph Definition

A Graph G=(V,E) consists of a V, a set of vertices that are not empty and E, a set of edges. Each edge has between one or two points connected to it, called and points [5].

Two vertices on non-directed graph G is said to be neighbors if the two are connected directly to a side. In other words, u adjacent to v if (u,v) is an edge in a graph degree of a node in non-trending graph is the number of side by side with that node [6].

Varieties of Graph

Graphs can be grouped into several categories (types) depending on the viewpoint of the grouping. Graph grouping can be seen based on the presence or absence of double sides or side edges or based on direction orientation on the side [4].
1. Simple graph and Unsimple graph
2. Directed graph and Undirected graph

Basic Terminology
1. Adjacent and Incident

Two vertices v and w are said to be Adjacent when both are connected directly to an e=(v,w), and side e=(v.w) is said to be incident or directly related to vertices v and w [7].
2. Degree

The degree of a vertex v of G is the number of edges incident with v, and is written $\deg(v)$; in calculating the degree of v, usually make the convention that a loop at v contributes 2 (rather than 1) to the degree of v [7].
3. Connected and Disconnected Graph

Suppose that G is a graph, then every two vertices v and w in G said to be connected in and only if there is

a path from vertex v to w or w to v. Graph G is said to be disconnected if and only if there are two vertices in G which are disconnected [8].

Planar Graph

According to Wilson [7] planar graph is a graph that can be drawn in the plane without crossings, so that is no two edges intersect geometrically except at a vertex to which both are incident. Any such drawing is a plane drawing. For convenience, we often use the abbreviation plane graph for a plane drawing of a planar graph. For example, figure 1 shows two drawing of the planar graph K4.
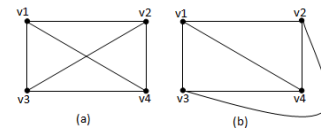

Figure 1(a). Planar Graph **(b)**. Plane Graph

Dual Graph

Dual graph is a planar graph G which is represented as a plane graph, so a graph of G* can be made which is geometrically is dual from planar graph in the following way [9].
1. Every area or face in G, make a vertex which is a vertex for G*.
2. Every edge e in G, draw the edge e* (which is the edge of G*) that intersect the edge of e. The edge e* connects two vertices v1* and is separated by the edge e in G.
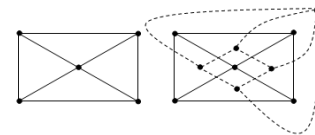

Figure 2. Dual graph formation

Graph Coloring

In graph theory, graph coloring is a form of graph labeling, that is by giving color to graph elements which will be subject to understanding problem constraints. There are three types of graph coloring problems that is vertex coloring, edge coloring, and region coloring [10].

Map Coloring

Some principles must be considered in coloring maps that are [11]:
1. The number of colors that used must be as minimum as possible.
2. Coloring the areas on the map means coloring the vertices in the graph.
3. Two vertices that connected by one or more edges cannot be given the same color.
4. In coloring the map, use an optimum color, meaning the new color will be used if the first color cannot be used again.

Sequential Color Algorithm

The sequential color algorithm is an algorithm for coloring a graph with k-color, where k is a positive integer. The method that used by this algorithm is by directly coloring a graph with as little color as possible [1]. The followings are the steps of the sequential color algorithm:

1. $G = (V, E)$ is a graph with the number of vertices are v. give the name of the vertices graph with $x_1, x_2, x_3, \dots, x_v$.
2. Create $L_i = <1,2,3,\dots,v>$, with L_i is set of colors that might be the color of the vertex x_i, starting from $i = 1$ to v.
3. Perform coloring sequentially based on the sequence of vertices x_i, starting from $i = 1$ to v, with the following method:
   3.1. Color the vertex x with C_i (C_i is the first color in the list L_i).
   3.2. For $j = 1$ to v, do:
      If $(x_i, x_j) \in E(G)$ then $L_j = L_j - C_i$, the meaning is if C_i member of L_j, throw away C_i from L_j, cause x_j may not be colored with C_i color, because C_i has become the color of x_i that adjacent with x_j. L_j is set of colors that might be a color from x_j.
4. Each vertex has been colored and the number of colors used is calculated.

2. RESEARCH METHODOLOGY

Time and Research Place

This research has been done at Advanced Computer Laboratory Department of Mathematics, Faculty of Mathematics and Natural Sciences, Sam Ratulangi University from December 2018 until May 2019.

Research Methods

The method that used in this research is a literature study by studying the literature related to the graph and its coloring and the sequential color algorithm.

Research Steps

The steps to be used in completing this research are as follows:

1. Understanding graph terminology.
2. Understanding graph coloring.
3. Accessing and observing the map of Minahasa Regency, North Sulawesi Province with the boundaries of each district.
4. Making a dual graph from the map of Minahasa Regency.
5. Applying the Sequential Color Algorithm to do region coloring on the map of Minahasa Regency.
6. Determining the minimum color $\chi(G)$ that used to color the map of Minahasa Regency.

3. RESULT AND DISCUSSION

Minahasa Regency

Figure 3. The map of Minahasa Regency and its sub-districts

Minahasa Regency consists of 25 sub-districts in it, which are regency with the highest number of sub-districts in North Sulawesi Province.

To simplify the application of graph theory into the map of Minahasa Regency, so the map of Minahasa Regency represented into a graph where the sub-districts in the Minahasa Regency map are assumed as a vertex and all the boundaries are assumed to be edges:

x_1 : Tombari Sub-district
x_2 : Tombari Timur Sub-district
x_3 : Pineleng Sub-district
x_4 : Mandolang Sub-district
x_5 : Tombulu Sub-district
x_6 : Tondano Utara Sub-district
x_7 : Tondano Barat Sub-district
x_8 : Tondano Selatan Sub-district
x_9 : Remboken Sub-district
x_{10} : Sonder Sub-district
x_{11} : Kawangkoan Utara Sub-district
x_{12} : Kawangkoan Sub-district
x_{13} : Kawangkoan Barat Sub-district
x_{14} : Tompaso Sub-district
x_{15} : Tompaso Barat Sub-district
x_{16} : Langowan Barat Sub-district
x_{17} : Langowan Utara Sub-district
x_{18} : Langowan Timur Sub-district
x_{19} : Langowan Selatan Sub-district
x_{20} : Kakas Barat Sub-district
x_{21} : Kakas Sub-district
x_{22} : Eris Sub-district
x_{23} : Lembean Timur Sub-district
x_{24} : Kombi Sub-district
x_{25} : Tondano Timur Sub-district

Figure 4. Representation of Minahasa Regency into a Graph

Dual Graph of Minahasa Regency's Map

Dual graphs from the map of Minahasa regency are made by representing existing sub-districts as vertices and neighboring vertices connected by edges.
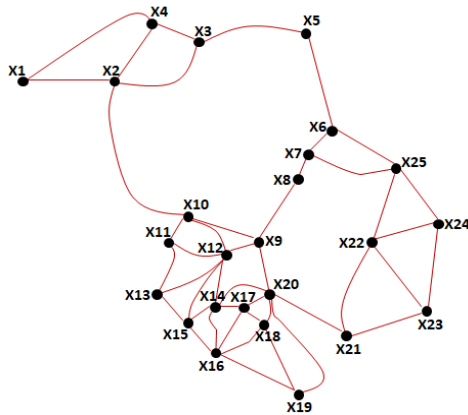

Figure 5. Dual graph from the Minahasa Regency's map

Region Coloring of Minahasa Regency's Map uses The Sequential Color Algorithm

Region coloring on the Minahasa regency's map is done by using the concept of region coloring, which is to color each region that neighboring with a different color. The following will explain how the sequential color algorithm works for coloring the Minahasa regency map.

Step 1: Determine the set of vertices and edges in the graph. Graph $G = (V, E)$ is a graph consisting of set vertices and edges. if the vertex on the graph has not been named, then the graph must be named first, for example by the name $x_1, x_2, x_3, \ldots, x_v$. then suppose the colors that might color the vertices are $1,2,3,\ldots,v$.

Based on figure 11(a), a dual graph that formed from the results of graph representation on the map of Minahasa regency has 25 vertices. The graph vertices are $x_1, x_2, x_3, \ldots, x_{25}$.

$G = (V, E)$
$V = \{x_1, x_2, x_3, x_4, x_5, \ldots, x_{25}\}$
$E = \{(x_1, x_2), (x_1, x_4), (x_2, x_3), (x_2, x_4), (x_2, x_{10}),$
$(x_3, x_4), (x_3, x_5), (x_5, x_6), (x_6, x_7), (x_6, x_{25}), (x_7, x_8),$
$(x_7, x_{25}), (x_8, x_9), (x_9, x_{10}), (x_9, x_{12}), (x_9, x_{20}), (x_{10}, x_{11}),$
$(x_{10}, x_{12}), (x_{11}, x_{12}), (x_{11}, x_{13}), (x_{12}, x_{13}), (x_{12}, x_{14}),$
$(x_{12}, x_{15}), (x_{13}, x_{15}), (x_{14}, x_{15}), (x_{14}, x_{16}), (x_{14}, x_{17}),$
$(x_{14}, x_{20}), (x_{15}, x_{16}), (x_{16}, x_{17}), (x_{16}, x_{18}), (x_{16}, x_{19}),$
$(x_{17}, x_{18}), (x_{17}, x_{20}), (x_{18}, x_{19}) \ (x_{18}, x_{20}), (x_{19}, x_{20}),$
$(x_{20}, x_{21}), (x_{21}, x_{22}), (x_{21}, x_{23}), (x_{22}, x_{23}), (x_{22}, x_{24}),$
$(x_{22}, x_{25}), (x_{23}, x_{24}), (x_{24}, x_{25})\}$

Step 2: Determine L_i, that is a set of color which might be the color of the vertex x. Suppose the colors that might color the vertices of the graph are $1,2,3,\ldots,25$. All the vertices of the graph are given different colors first so that obtained L_i for the vertices of the graph in figure 11(b) are as follows:

L_1 for x_1 is $\{1\}$
L_2 for x_2 are $\{1,2\}$
L_3 for x_3 are $\{1,2,3\}$
L_4 for x_4 are $\{1,2,3,4\}$
L_5 for x_5 are $\{1,2,3,4,5\}$
L_6 for x_6 are $\{1,2,3,4,5,6\}$
L_7 for x_7 are $\{1,2,3,4,5,6,7\}$
L_8 for x_8 are $\{1,2,3,4,5,6,7,8\}$
L_9 for x_9 are $\{1,2,3,4,5,6,7,8,9\}$

L_{10} for x_{10} are $\{1,2,3,4,5,6,7,8,9,10\}$
L_{11} for x_{11} are $\{1,2,3,4,5,6,7,8,9,10,11\}$
L_{12} for x_{12} are $\{1,2,3,4,5,6,7,8,9,10,11,12\}$
L_{13} for x_{13} are $\{1,2,3,4,5,6,7,8,9,10,11,12,13\}$
L_{14} for x_{14} are $\{1,2,3,4,5,6,7,8,9,10,11,12,13,14\}$
L_{15} for x_{15} are $\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15\}$
L_{16} for x_{16} are $\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16\}$
L_{17} for x_{17} are $\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,\ldots,17\}$
L_{18} for x_{18} are $\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,\ldots,18\}$
L_{19} for x_{19} are $\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,\ldots,19\}$
L_{20} for x_{20} are $\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,\ldots,20\}$
L_{21} for x_{21} are $\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,\ldots,21\}$
L_{22} for x_{22} are $\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,\ldots,22\}$
L_{23} for x_{23} are $\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,\ldots,23\}$
L_{24} for x_{24} are $\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,\ldots,24\}$
L_{25} for x_{25} are $\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,\ldots,25\}$

Step 3: Do the sequential coloring based on the sequence of x_i vertices, starting from $i = 1$ to v, in the following way:

- **Step 3.1 :** Color the vertex x_i with C_i (C_i is the first color in list L_i)
- **Step 3.2** : For $j = i$ to v, do:
  if $(x_i, x_j) \in E(G)$ then $L_j = L_j - C_i$ (where C_i here is the first color in list L_j), that is if C_i is the member of L_j, eliminate C_i from L_j, because x_j cannot be colored by C_i, cause C_i has become the color of x_i that is adjacent with x_j. L_j is the set of colors that might be the color of x_j.

Based on the result of coloring using sequential color algorithm, so the number of colors that obtained from the coloration are 3 colors as follows:

1. Vertex x_1 colored by color 1
2. Vertex x_2 colored by color 2
3. Vertex x_3 colored by color 1
4. Vertex x_4 colored by color 3
5. Vertex x_5 colored by color 2
6. Vertex x_6 colored by color 1
7. Vertex x_7 colored by color 2
8. Vertex x_8 colored by color 1
9. Vertex x_9 colored by color 2
10. Vertex x_{10} colored by color 1
11. Vertex x_{11} colored by color 2
12. Vertex x_{12} colored by color 3
13. Vertex x_{13} colored by color 1
14. Vertex x_{14} colored by color 1
15. Vertex x_{15} colored by color 2
16. Vertex x_{16} colored by color 3
17. Vertex x_{17} colored by color 2
18. Vertex x_{18} colored by color 1
19. Vertex x_{19} colored by color 2
20. Vertex x_{20} colored by color 3
21. Vertex x_{21} colored by color 1
22. Vertex x_{22} colored by color 2
23. Vertex x_{23} colored by color 3
24. Vertex x_{24} colored by color 1
25. Vertex x_{25} colored by color 3

If for example color 1 is red, color 2 is green, and color 3 is yellow, so the colored graph is as follows:
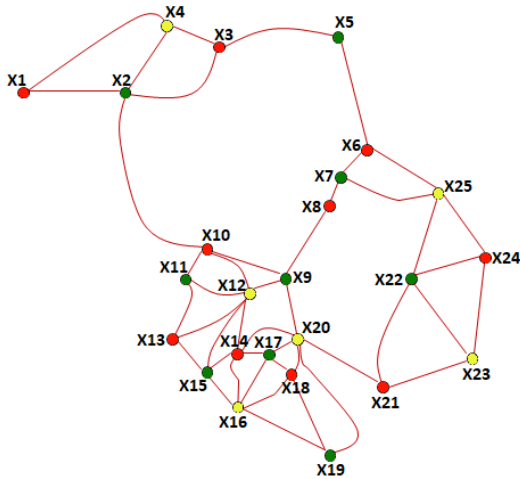
Figure 6. The colored graph

Determine The Minimum Number of colors of Minahasa Regency Map

The minimum number of colors or called chromatic numbers $\chi(G)$ obtained from the results of coloring the sub-district of the Minahasa regency map using the sequential color algorithm can be seen from the number of colors that needed in the graph coloring. According to the concept of region coloring, vertices on dual graphs of graphs that representing maps of Minahasa regency represent existing sub-districts, so that the colors used for a vertex mean colors that can be used for coloring the region they represent.

Based on the results obtained from the vertex coloring of the dual map of the Minahasa regency map using a sequential color algorithm, the minimum color obtained is 3 colors. So the colors needed to color the 25 sub-districts in Minahasa regency only need 3 colors ($\chi(G) = 3$) in the coloring process.
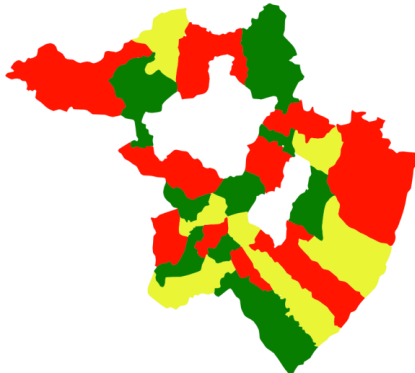

Figure 7. Map of Minahasa Regency which all the sub-districts has been colored

As you can see that the color that becomes the color of the existing sub-district is the node color in the previous dual graph, because it is in accordance with the concept of regional coloring, each color that becomes the color of the sub-district on the map is represented by the color of the vertices on the second part representing each sub-district.

4. CONCLUSIONS AND SUGGESTIONS

Conclusions

Based on the results obtained, it can be concluded that:

1. Region Coloring in the map of Minahasa Regency can be done using The Sequential Color Algorithm by creating a dual graph form of the Minahasa Regency's Map.
2. The Minimum number of colors or chromatic numbers $\chi(G)$ obtained in the coloring of 25 sub-districts on the map of Minahasa Regency in this research obtained 3 colors ($\chi(G) = 3$), with colors between neighboring regions having different color.

Suggestions

This final project discusses one of the application in the field of graph theory about graph coloring which is applied to the coloring of regions on the map using Sequential Color Algorithm. Another research that can be developed from this study is that graph theory coloring can also be done on wider maps with different algorithms and using mathematical software or applied in another cases that has been explained in Chapter before.

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Yevie Ingamita (yevie13@gmail.com)

Born in Lembor, East of Nusa Tenggara, Indonesia on May 13th, 1997. Took her college in Mathematics Department, Faculty of Mathematics and Natural Sciences, University of Sam Ratulangi Manado. In 2019 was the last year she took her bachelor's degree. This paper is the result of her research that published.

Nelson Nainggolan (n-nelson@unsrat.ac.id)

Born on March 9th, 1967. In 1992 he received a Bachelor of Science (S.Si.) degree from Medan State University. He obtained his Master of Science (M.Si.) degree at Institute of Technology Bandung in 1996. He is an Associate Professor at Faculty of Mathematics and Natural Sciences, Univerty of Sam Ratulangi.

Benny Pinontoan (bpinto@yahoo.com)

Born in Bitung, North Sulawesi, Indonesia, and lives in Manado. Completed his bachelor's degree in Technische Informatica at Faculteit Informatica Eindhoven, Netherlands in 1993. In 2002 he completed his Doctoral degree in Mathematics at School of Mathematics and Statistics, Carlethon University Ottwa Canada. In 1995 he was appointed lecture at Sam Ratulangi University (UNSRAT) and since 1 March 2006 became a Profesor of Mathematics at Faculty of Mathematics and Natural Sciences, UNSRAT.