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Crossing Number of Infinite Family of Extension Kochol's Periodic Graphs

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ABSTRACT

At this time, technology is developing very quickly and is increasingly sophisticated. This technological development is certainly closely related to the development of computer technology. A computer is able to control a series of electronic devices using an IC chip that can be filled with programs and logic called microprocessor technology. A microprocessor is a digital component of the VLSI (Very Large Scale Integration) type with very high circuit complexity that is capable of carrying out the functions of a CPU (Central Processing Unit). Among many applications, the problem of crossing number very interesting and important because of its application in the optimization of chip are required in a circuit layout of VLSI. Crossing number used to obtain the lower bound on the amount of chip area of VLSI devices like microprocessor and memory chips additionally, crossings in the circuit layout could cause short circuit and therefore worth minimized independent of the chip area consideration. Some graph can be seen as built by small pieces. A principal tool used in construction of crossing-critical graphs are tiles. In the tile concept, tiles can be arranged by gluing one tile to another in a linear or circular fashion. The series of tiles with circular fashion form an infinite graph family. In this way, the intersection number of this family of graphs can be determined. In this research, has been formed an infinite family graphs $Q(K_8)$. The graph formed by gluing together many copies of the tile $P(K_n)$ in circular fashion, where the tile $P(K_n)$ consists of identical tile sections. The results obtained show that the graph $Q(K_8)$ has 3-crossing-critical of a graph.

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ABSTRAK

Pada saat ini, teknologi berkembang dengan sangat cepat dan semakin canggih. Perkembangan teknologi ini tentunya sangat erat kaitannya dengan perkembangan teknologi komputer. Sebuah komputer mampu mengendalikan serangkaian perangkat elektronik dengan menggunakan sebuah chip IC yang dapat diisi dengan program dan logika yang disebut dengan teknologi mikroprosesor. Mikroprosesor adalah sebuah komponen digital berjenis VLSI (Very Large Scale Integration) dengan kompleksitas rangkaian yang sangat tinggi yang mampu menjalankan fungsi sebuah CPU (Central Processing Unit). Di antara banyak aplikasi, masalah bilangan persilangan sangat menarik dan penting karena aplikasinya dalam optimasi chip yang diperlukan dalam tata letak sirkuit VLSI. Angka persilangan digunakan untuk mendapatkan batas bawah pada jumlah area chip perangkat VLSI seperti mikroprosesor dan chip memori. Persilangan dalam tata letak sirkuit dapat menyebabkan korsleting dan oleh karena itu perlu diminimalkan terlepas dari pertimbangan area chip. Beberapa grafik dapat dilihat dibangun oleh potongan-potongan kecil. Alat utama yang digunakan dalam mengkonstruksi graf perpotongan kritis adalah ubin. Dalam konsep ubin, ubin dapat disusun dengan merekatkan satu ubin ke ubin lainnya secara linier atau melingkar. Rangkaian ubin yang disusun secara melingkar membentuk keluarga graf tak hingga. Dengan cara ini, angka perpotongan dari keluarga graf ini dapat ditentukan. Dalam penelitian ini telah dibentuk graf keluarga tak hingga $Q(K_8)$. Graf dibentuk dengan merekatkan banyak salinan ubin $P(K_n)$ secara melingkar, dimana ubin $P(K_n)$ terdiri dari sebuah ubin yang identik. Hasil yang diperoleh menunjukkan bahwa graf $Q(K_8)$ memiliki 3-perpotongan kritis.

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1. INTRODUCTION

At this time, technology is developing very quickly and is increasingly sophisticated. This technological development is certainly closely related to the development of computer technology. Where computer technology is a supporter and even a driver of the advancement of information technology today. And it cannot be denied that electronic science is very influential on the development of technology. A computer is able to control a series of electronic devices using an IC chip that can be filled with programs and logic called microprocessor technology. A microprocessor is a digital component of the VLSI (Very Large Scale Integration) type with very high circuit complexity that is capable of carrying out the functions of a central processing unit (CPU). A microprocessor, often called a CPU, is a control element in a computer system. The microprocessor controls memory and I/O (Input/Output) via a number of connection lines called buses. The bus selects memory or I/O devices, transfers data between I/O devices and memory with a microprocessor, and controls the I/O and memory systems. I/O and memory are controlled through instructions stored in memory and executed by the microprocessor [4].

Graph theory is crucial for managing and analyzing these pervasive systems. A graph is an effective tool for managing the complexity of large scale VLSI systems. By reducing the complex components of a VLSI system into nodes and edges. Classic graph theoretic problems, such as Steiner minimal trees, pathfinding, and graph partitioning, have been extensively studied and applied, in no small part, due to the practical effectiveness of graph theory to the design and analysis of VLSI systems. A virtuous cycle of theory and application has greatly advanced both graph theory and ever more powerful VLSI systems. Many areas of the VLSI design process, such as system exploration and the integration of emerging technologies, will require novel design methodologies and algorithms, many of which rely on graph theory [1].

A graph is a collection of points (or vertices) together with a set of edges (or curves) each of which joins two distinct vertices of the graph. The first graph theory was discovered by Swiss mathematician Leonhard Euler, solved the Königsberg bridge problem. Königsberg, known as Kaliningrad, was the capital of East Prussia. Through the city flows the river Pregel, which divides the city into four regions which were connected to each other with seven bridges. The Königsberg bridge problem was an old puzzle concerning the possibility of finding a path over every one of seven bridges that span a forked river flowing past an island but without crossing any bridge twice. Euler argued that no such path exists. His proof involved only references to the physical arrangement of the bridges, but essentially he proved the first theorem in graph theory.

The crossing number of a graph G is the minimum number of crossing points of edges in a drawing of G in the plane. Among many applications, the problem of crossing number very interesting and important because of its application in the optimization of chip are required in a circuit layout of VLSI (Very Large Scale Integration). Crossing number used to obtain the lower bound on the amount of chip area of VLSI devices like microprocessor and memory chips [6]; additionally, crossings in the circuit layout could cause short circuit and therefore worth minimized independent of the chip area consideration . Some graph can be seen as built by small pieces.

The tile concept was first introduced by Pinontoan and Richter (2003), in research about " Crossing Numbers of Sequences of Graph in Case Planar Tiles". A tile is a 3 tuple $T = (G, L, R)$ which is connected graph together with two specified sequences of vertices, the left walls (L) and right walls (R). A tile drawing of a tile T is a drawing of the unit square with the left wall in order down the left hand side and the right wall in order down the right hand side. Tiles can be arranged by gluing one tile to another and arranged in a linear or circular fashion. The tile T^n is obtained by gluing n copies of T in linear fashion, while the graph $\circ(T^n)$ is obtained by gluing n copies of T in a circular fashion. The series of tiles with circular fashion form an infinite graph family. In this way, the intersection number of this family of graphs can be determined.

In Pinontoan and Richter (2003) research, they describe a method of creating an infinite family of crossing-critical graphs from a single small planar map, the tile, by gluing together many copies of the tile together in a circular fashion.

In previous research, Kochol define a graph "Construction of Crossing-Critical Graphs", he proved that a graph is crossing-critical if deleting any edge decreases its crossing number on the plane. For any $n \geq 2$ present a construction of an infinite family of 3-connected crossing-critical graphs with crossing number n. In this research, we modify Kochol's graph Kn^* and use the theorem by Pinontoan and Richter to compute Kn^*. And then, will be seen the number of crossings on the graphs.

2. LITERATURE REVIEW

2.1 Microprocessor

A microprocessor is a chip that can carry out counting operations, reasoning operations, and control operations electronically (digitally). Control electronically (digital), usually the microprocessor is packaged with plastic or ceramic. The packaging is equipped with pins which are input and output terminals of the chip. Microprocessor is an integrated circuit (IC) in the form of a VLSI (Very Large Scale Integration) chip component which is able to execute commands sequentially in form of program so that it can work as desired programmer. The command or instruction given to a microprocessor must be understood by the microprocessor itself. In general, instructions are given in the form of binary quantities or in machine language.

Each microprocessor has a different instruction code according to the manufacturer's plan. So that a program written in the instruction code for a particular microprocessor cannot be run for all types of microprocessors that exist [4].

2.2 VLSI (Very Large Scale Integration)

VLSI (Very Large Scale Integration) design is a process of designing integrated circuits (IC) by integrating thousands, millions or even billions of transistors on a single chip. The history of VLSI design can be traced back to the 1950s when the first transistor was invented. Later on, in the 1960s, the first integrated circuit (IC) was developed which revolutionized the electronics industry. With the development of ICs, the size of electronic devices reduced drastically, and their functionality increased. In the 1970s, the first microprocessor was invented, which gave birth to the modern computer era. The 1980s saw the emergence of VLSI design as a discipline, and the first VLSI chip was designed in 1983. Since then, VLSI design has been advancing at a rapid pace, and the technology has evolved to produce more complex and efficient chips. Two of the most common VLSI devices are the microprocessor and the microcontroller.

The VLSI physical design process manipulates very simple geometric objects, such as polygons and lines. As a result, physical design algorithms tend to be very intuitive in nature, and have significant overlap with graph algorithms and combinatorial optimization algorithms. In view of this observation, many consider physical design automation the study of graph theoretic and combinatorial algorithms for manipulation of geometric objects in two and three dimensions [14].

2.3 Graph Theory

Graph G is defined as a set pair (V, E), written with the notation $G = (V, E)$, in which case V is a nonempty set of vertices or nodes and E is the set of edges or arcs connecting a pair of vertices [8]. According to history, graph theory became known when a Swiss mathematician named Leonhard Euler, successfully revealed the mystery of the Königsberg bridge in 1736. In the city of Königsberg (now called Kaliningrad, in the Soviet Union) flows a river called Pregel river. In the middle of the river were two islands. From both islands, there was a bridge that reached the river bank and between the two islands.

A linear graph (or a simple graph) $G = (V, E)$ consists of a group of objects $V = \{v_1, v_2, \dots v_n\}$ called a set of points, and another set of $E = \{e_1, e_2, \dots . e_n\}$ which is a set of sides such that each side of e_k is associated with an unordered pair (v_i, v_j). Points v_i, v_j related to e_k are called the end points of the $e_k$side. The most common way to represent a graph is in the form of a diagram. In the diagram, the points are expressed as dots and each side is expressed as a line segment connecting each two points [15].

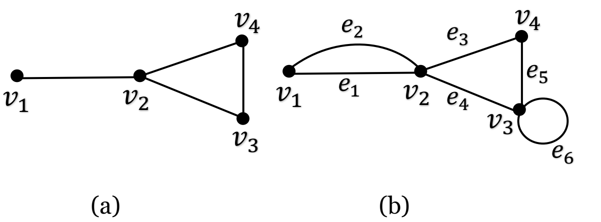


Figure 1. (a) Simple Graph; (b) Non Simple Graph (Multigraph)

2.4 Crossing Number

The crossing number of a graph G is the smallest number of pairwise crossings of edges among all drawings of G in the plane [13]. For $k \in n$, a graph G is called k-crossing-critical, if $cr\ (G) \geq k$ but, for every edge e of G, $cr\ (G - e) < k$.

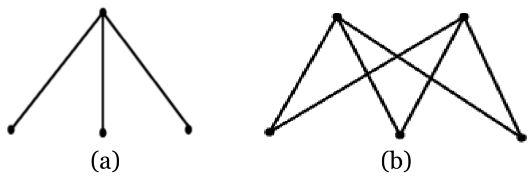


Figure 2. Crossing Number of; (a) $K_{1,3}(Cr_{1,3})$= 0, (b) $K_{2,3}(Cr_{2,3})$= 3

In general, for a graph G, the minimum number of pairwise crossings of edges among all drawings of G in the plane is the crossing number of G and is denoted by $cr\ (G)$. Thus, $cr_{2,3} = 3$. We remark that $cr_{2,3} \geq 3$. It is an interesting exercise for the reader to prove that $cr_{2,3} = 3$. At present, there is no known efficient algorithm to calculate the crossing number of an arbitrary graph. In fact, the problem of calculating the crossing number of a graph is NP-complete, so it is unlikely that such an efficient algorithm exists. Yet one might hope that the crossing number of a graph with special structure can be calculated [12].

According to [9] a graph can be drawn in many ways on the plane. A good drawing has the following properties :

– No edge crosses itself
– No pair of adjacent edges cross
– Two edges cross at most once
– No more than two edges cross at the point

2.5 Crossing-Critical Graph

A graph G is k-crossing-critical if $cr\ (G) \geq k$ and $cr\ (G - e) < k$, for any edge e of G. It is important to study crossing critical graphs in order to understand structural properties of the crossing number problem. The only 1-crossing critical graphs are subdivisions of K_5 and $K_{3,3}$ [13].

According to Kuratowski's theorem, a graph G is nonplanar if and only if G has a subgraph that's a subdivision of K_5 and $K_{3,3}$. To prove that a graph was planar by looking at all subgraphs of it and showing none of them looked like K_5 and $K_{3,3}$. But if a graph G s nonplanar, we can always use Kuratowski's theorem to prove that it's nonplanar. The tricky part of using Kuratowski's theorem is actually finding the desired subgraph. It will be possible to do so by educated trial an error. A few rules of thumb may be helpful, however. First, note that subdivision cannot increase the degree of any vertex. So, for G to have a subgraph that's a subdivision of K_5, G has to have at least 5 vertices with degree at least 4.

Conceptually, it can be useful to think that some vertices of G are going to be vertices of your K_5 and $K_{3,3}$, and we're going to need to connect those. We can use the remaining vertices of G as parts of subdivided edges between these, but these extra vertices can only be used in at most one such connection. Thus, these extra vertices are a limited resource we have, and a useful heuristic in looking for subgraph is to take a "greedy" approach, where we choose our vertices to require as few subdivisions as possible to make connections.

2.6 Tile

Pinontoan and Richter (2003), have tile concepts and terminology. A tile is a 3-tuple $T = (G, L, R)$, where

- G is a connected graph;
- L is a finite sequence of vertices of $V(G)$, called the left-wall;
- R is a finite sequence of vertices of $V(G)$, called the right-wall; and
- And all the vertices in L and R are distinct.

Figure 3. shows some examples of tiles, the tiles K, M, and Z. The left-wall of each of these tiles is the sequence (a, b, c) and right-wall is the sequence (d, e, f) of vertices [10].

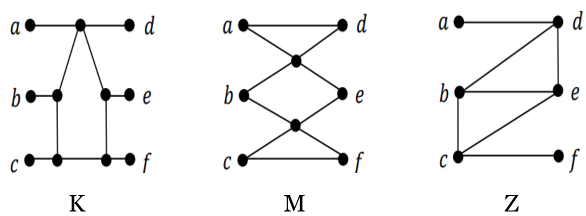


Figure 3. Tile K, M, and Z

A tile drawing of a tile $T = (G, L, R)$ is a drawing of T in the unit square S = {(*x*,*y*): -1 ≤ *x* ≤ 1 and −1 ≤ *y* ≤ 1} such that: the intersection with the boundary {(*x*,*y*) ∈ *S*: either *x* ∈ {-1,1} or *y* ∈ {-1,1}} is precisely $L \cup R$; the vertices in L occur in the line x = -1, with the y-coordinates of $L[1], L[2], \ldots, L[|L|]$ decreasing; and the vertices in R occur in the line x = 1, with y-coordinates of $R[1], R[2], \ldots, R[|R|]$ also decreasing. The tile crossing number *tcr* (T) of a tile T is the smallest number of crossings in any tile drawing of T [11].

If $tcr(T) = 0$, a tile $T = (G, L, R)$ is planar, and otherwise the tile is non-planar. The twist $T' = (G, L, R')$ of $T = (G, L, R)$ is the tile obtained by reversing the order of R. A tile $T_1 = (G_1, L_1, R_1)$ is compatible with a tile $T_2 = (G_2, L_2, R_2)$ if the function $f\colon R_1 \rightarrow L_2$ defined by $f\ (R_1[i] = L_2[i]$ is bijection and $\forall v_1, v_2 \in R_1, v_1, v_2 \in E(G) \Leftrightarrow f(v_1) f(v_2) \in E(G_2)$. A tile is self-compatible if it is compatible to itself [11].

3. RESEARCH METHODOLOGY

3.1 Time and Research Place

This research has been held in Mathematics Departments of Sam Ratulangi University from April until Oktober 2023.

3.2 Research Methods

This research had use the literature study method from books and journals. From the literature study we can see, learn, showing existing result and develops it in graph theory.

3.3 Research Stage

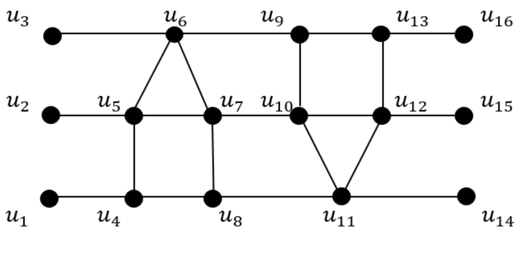
The steps to be used in completing this research are as follows:

1. Study the literature from books and journals relating to the problem.
2. Make construction of crossing number of infinite family of extension Kochol's periodic graphs using the tile concept.
3. Determine $cr({K_n}^*)$ of continuing periodic graphs by Kochol's.

4. RESULTS AND DISCUSSION

4.1 Construction of Tile

In this research, the tile concept is very important and is used to construct an infinite family of graphs. Therefore, the first thing to do is to construct the tiles.



S

Figure 4. Tile S

Figure 8 shows the tile $S = ((\{ u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}, u_{15}, u_{16} \}, \{ u_1u_4, u_4u_8, u_4 u_5, u_2u_5, u_5u_7, u_5u_6, u_3u_6, u_6u_7, u_6u_9, u_7u_{10}, u_7u_8, u_9u_{13}, u_9u_{10}, u_{10}u_{12}, u_{10}u_{11}, u_{11}u_{12}, u_{11}u_{14}, u_{12}u_{15}, u_{12}u_{13}, u_{13}u_{16}\})$,$(u_6, u_7, u_8), (u_9, u_{10}, u_{11}))$. A tile $S = (G_1, L_1, R_1)$, A tile is self-compatible if it is compatible with itself. If S be a self-compatible tile, then S_n is the catenation of n copies of S in linear fashion and $\circ(S_n)$ is the catenation of n copies of S in circular fashion where the first and the last order of tile S also put in catenation.

Tile $S = ((\{ u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}, u_{14}, u_{15}, u_{16} \}, \{ u_1u_4, u_4u_8, u_4u_5, u_2u_5, u_5u_7, u_5u_6, u_3u_6, u_6u_7, u_6u_9, u_7u_{10}, u_7u_8, u_9u_{13}, u_9u_{10}, u_{10}u_{12}, u_{10}u_{11}, u_{11}u_{12}, u_{11}u_{14}, u_{12}u_{15}, u_{12}u_{13}, u_{13}u_{16} \}), (u_6, u_7, u_8), (u_9, u_{10}, u_{11}))$. The tile S are planar, while tile S' are not planar. For the twist S' has tile crossing number $tcr(S')$ = 3. According to the rules of crossing number, to get the smallest crossing number, the twist can be through the tile S.

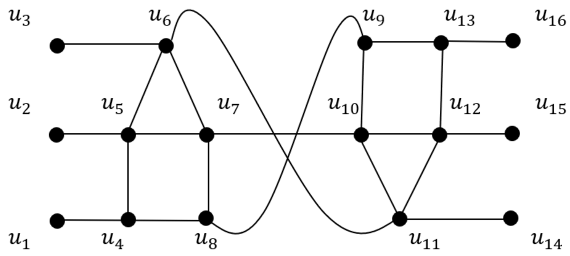


Figure 5. Tile S'

4.2 Infinite Family of Extension Kochol's Periodic Graphs

In this research, building an infinite family of graphs, it is all taken and refers to the tile theory that has been introduced by Pinontoan and Richter. They introduced the theory of tile which is two small pieces of graph with certain conditions and can be glued together in a circular fashion to built an infinite family of graphs. Kochol define the infinite family for any n ≥ 2 there is an infinite family of 3-connected crossing-critical simple graphs with crossing number n. In this research, will be set *edge* = $(u_5u_7, u_{10}u_{12})$ and define k ≥ 0, for the tile P (K_n) is a tile consisting of tile S in Figure 11, and added with *edge* = (u_2u_4, u_7u_9) in linear fashion.

4.2.1 Tile $P(K_n)$

For the example will define K_n = 8, thus forming a tile $P(K_8)$ in linear fashion (Figure 10). The tile P are planar, where without crossing. To get crossing, the order of R in tile P must be reversed. If the twist is through the tile S, the tile has 3-crossing-critical (Figure 11). Therefore, according to the rules of tile crossing number, the tile smallest number of crossing is used, which is the twist through the tile S.

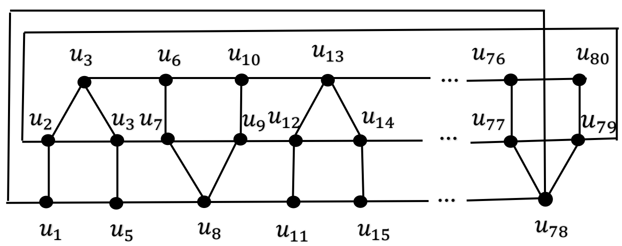


Figure 6. $P'_{(K_8)}$

4.2.2 The Infinite Family Graph $Q(K_n)$

For $k \geq 0$, the graph $Q\,(K_n) = \circ(P\,'P\,^{n-1})$ has vertices $u_{1m},\ u_{2m},\ u_{3m},\ u_{4m},\ u_{5m},\ u_{6m},\ u_{7m}, u_{8m},\ u_{9m}, u_{10m}, \cdots,\ u_{1sm},\ u_{2sm}, u_{3sm}, u_{4sm}, u_{5sm}, u_{6sm}, u_{7sm}, u_{8sm},\ u_{9sm},\ u_{10sm},$ for $(1 \leq m \leq n)$ and the edges $u_{1km}u_{2km},\ u_{1km}u_{5km},\ u_{2km}u_{3km},\ u_{3km}u_{4km},\ u_{4km}u_{5km},\ u_{4km}u_{7km},\ u_{7km}u_{6km},\ u_{7km}u_{8km}, u_{8km}u_{9km}, u_{9km}u_{10km},\ u_{6km}u_{1okm}$ $(1 \leq k \leq s)$ $(1 \leq m \leq n)$.

Pinontoan and Richter identified K_n as a graph made up by gluing n planar tiles K in Figure 4, and a single tile K' in Figure 5 which is the twist of K and isomorphic to K, in circulfashion, that is, $K_n = K^{\otimes}(n) = o(K^nK')$, and rename the vertices with appropiate names. Figure 7 shows K_8.

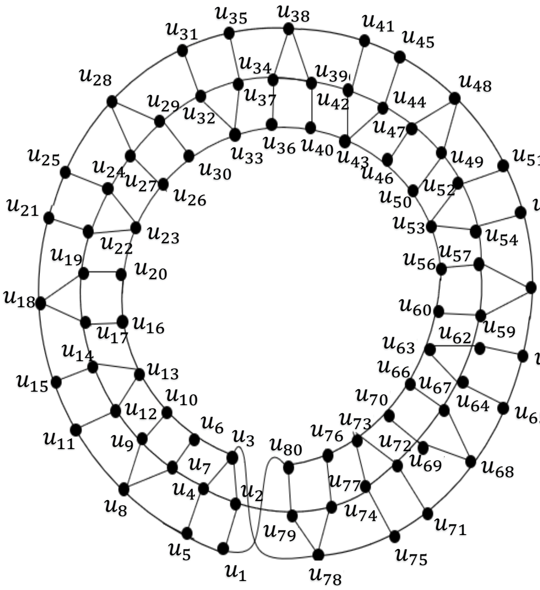


Figure 7. Graph $Q(K_8)$

5. CONCLUSION AND SUGGESTION

5.1 Conclusion

Based on the result of the study, it can be concluded that:

The infinite family of extension Kochol's periodic graphs has 3-crossing-critical. And the shape of infinite family graphs $Q\,(K_8)$ for added with *edge* = (u_2u_4, u_7u_9) the graph $Q\,(K_8) = \circ\ (P'\,P^{n-1})$ has vertices $u_{1m}, u_{2m}, u_{3m}, u_{4m}, u_{5m}, u_{6m}, u_{7m}, u_{8m}, u_{9m}, u_{10m}, \cdot\cdot, u_{1sm}, u_{2sm}, u_{3sm}, u_{4sm}, u_{5sm}, u_{6sm}, u_{7sm}, u_{8sm}, u_{9sm}, u_{10sm}$ for $(1 \leq m \leq n)$ and the edges $u_{1km}u_{2km}, u_{1km}u_{5km},\ u_{2km}u_{3km},\ u_{3km}u_{4km},\ u_{4km}u_{5km},\ u_{4km}u_{7km},\ u_{7km}u_{6km},\ u_{7km}u_{8km}, u_{8km}u_{9km}, u_{9km}u_{10km}, u_{6km}u_{10km}$ $(1 \leq k \leq s)$ $(1 \leq m \leq n)$.

5.2 Suggestion

Hopefully, through this research, the researchers can further develop theories about graphs especially tile concept and find the structure of infinite family graphs.

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Born in Samarinda, on April 19rd, 2000. She pursued a Bachelor's program in the Mathematics Department, Faculty of Mathematics and Natural Science in Sam Ratulangi University. 2023 is the last year she studied. This paper is the result of a published thesis she researched.

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