Jurnal Sipil Statik Vol.6 No.1 Januari 2018 (13-20) ISSN: 2337-6732

TRAFFIC FLOW MODEL AND SHOCKWAVE ANALYSIS

Semuel Y. R. Rompis
Fakultas Teknik, Jurusan Teknik Sipil, Universitas Sam Ratulangi Manado
e-mail: semrompis@fulbrightmail.org

Abstract

The flow, density and speed relationship is one among other most important element in traffic flow theory. There are many models that express the relationship between these three traffic stream primary elements. The first three models which have been used most commonly in traffic engineering practice are Greenshields, Greenberg and Underwood models. This study has built traffic flow model base on those models and decided the best model to represent the field data. The characteristic of the traffic flow will be obtained based on the chosen model, then using this information and an incident scenario a macroscopic shockwave analysis was conducted. The result of analysis, particularly the queue length, was compared to one obtained from simulation built in VISSIM.

Key words: traffic flow, density, speed, macroscopic shockwave analysis, VISSIM

INTRODUCTION

Background

Greenshields, Greenberg and Underwood models are the most commonly used models to state the relationships between flow, density and speed. These models are key factors for traffic engineers to explore the characteristic of traffic flow. One among other role of these relationships is to become a primary data for macroscopic shockwave analysis. This study utilized the field data to establish the Greenshields, Greenberg and Underwood traffic flow model. The characteristic derived from the best model become essential data to carry out the macroscopic shockwave analysis. The result from this analysis is then compared to the result of the one generated from simulation build in VISSIM.

The objective of the study

This study is expected to meet this objectives :

- Build the Greenshields, Greenberg and Underwood traffic model using the field data
- Determine a model that best represents the data
- Conduct macroscopic shockwave analysis under several scenario using characteristic of the best model as essential data
- Compare the result of macroscopic shockwave analysis to the microscopic one which is built from transportation software "VISSIM".

LITERATURE REVIEW

Mathematical relationship between traffic volume, speed and density

To obtain the traffic characteristic, one would need to know the mathematical relationship between flow, density and speed in a certain road segment. The relationship between speed, flow and density can be represented as equation 1 :

$$q = u.k \tag{1}$$

This relationship can be described using figure 1 (see [1] [2]) that shown the common relationship between speed and density (u-k), flow and density (q-k) and flow and speed (q-u).

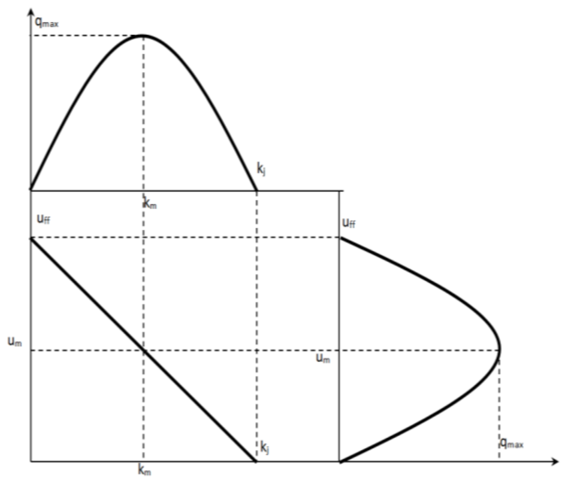


Figure 1. Fundamental Diagram

The fundamental diagram between u and k shows that as the density increase the speed will decrease. The flow will be 0 (zero) when the density is very high that it is not possible for the vehicle to move anymore. This situation is identified as jam density ($k=k_j$). On the other hand, when the density is 0, the flow is also 0 because there is no vehicle in the road (q=0). The characteristic between these two extreme points is the most essential that need to be known for traffic flow analysis.

As the density increases (from 0) the speed will decrease while the flow will increase. This will keep happen until it reaches a condition where the flow reach maximum and the increase of density will result on flow decrease. Such condition is known as flow capacity. Figure 1 also shows some essential parameter, defined as follows : q_m is capacity or maximum flow (vph), u_m is speed on flow capacity (mph), k_m is density on flow capacity (vpm), k_j is jam density (vpm) and u_{ff} is free flow speed (mph).

Data that can be observed from the field, by conducting traffic survey or by detector, are traffic flow and speed. In this study, there are 3 kinds of traffic flow model that were used to represent the mathematical relationship between the traffic flow, speed and density, which are, Greenshields model, Greenberg model and Underwood model

Greenshields Model

Greenshield formulated the mathematical relationship between speed and density as linear function [3], shown in equation 2 :

$$u = u_{ff} - \frac{u_{ff}}{k_j} k \quad (2)$$

Using equation (1), the relationship between flow and density was formulated and then by substituting (3) to (2), one can get the equation (4) and (5)

$$u = \frac{q}{k} \quad (3)$$

$$\frac{q}{k} = u_{ff} - \frac{u_{ff}}{k_j} k \quad (4)$$

$$q = k.u_{ff} - \frac{u_{ff}}{k_j} k^2 \quad (5)$$

The equation (5) express the relationship between flow and density. Maximum Flow (q_m) will be reached at the condition $k=k_m$, that is stated by (6) to (7).

$$\frac{\partial q}{\partial k} = u_{ff} - \frac{2.u_{ff}}{k_j} k_m = 0 \quad (6)$$

$$k_m = \frac{k_j}{2} \quad (7)$$

By substituting (7) to (5), the maximum flow (q_m) is shown in equation (8),

$$q_m = \frac{k_j.u_{ff}}{4} \quad (8)$$

The relationship between flow and speed can be formulated using equation (1) and by substituting (9) to (2), one can written equation (10) – (12)

$$k = \frac{q}{u} \quad (9)$$

$$u = u_{ff} - \frac{u_{ff}}{k_j} . \frac{q}{u} \quad (10)$$

$$\frac{u_{ff}}{k_j} . \frac{q}{u} = u_{ff} - u \quad (11)$$

$$q = k_j.u - \frac{k_j}{u_{ff}} u^2 \quad (12)$$

Equation 12 stated the relationship between flow and speed. The maximum flow (q_m) is reached when $u=u_m$ as expressed in (13)-(14).

$$\frac{\partial q}{\partial u} = k_j - \frac{2.k_j}{u_{ff}} . u_m = 0 \quad (13)$$

$$u_m = \frac{u_{ff}}{2} \quad (14)$$

By substituting (14) to (12), the q_m can be formulated as shown in (15) :

$$q_m = \frac{k_j.u_{ff}}{u} \quad (15)$$

Thus, can be concluded that q_m can be reach when $u=u_m$ and $k=k_m$.

Greenberg Model

Greenberg assumed the relationship between speed and density as logarithmic function (instead of linear function)[4] [3],

$$u = c \ln\left(\frac{k_j}{k}\right) \quad (16)$$

The equation (16) can be written as in (17), which is the speed and density relationship function :

$$u = c \ln k_j - c \ln k \quad (17)$$

The relationship between flow and speed can be formulated by substituting (17) to (1),

$$q = ck \ln k_j - ck \ln k \quad (18)$$

Equation (18) expresses the relationship between flow and density. Maximum flow (q_m) can be reached when $k=k_m$. $k=k_m$ is formulated in (22)-(24)

$$\frac{\partial q}{\partial k} = c \ln k_j - c(\ln k + 1) = 0 \quad (19)$$

$$\ln k_j = \ln k + 1 \quad (20)$$

$$\ln k = \ln k_j - 1 \quad (21)$$

$$k = k_j . e^{-1} \quad (22)$$

The flow speed relationship can be formulated using equation (1), by substituting (9) to (17) one can formulated the equation (23) – (27)

$$u = c \ln k_j - c \ln \frac{q}{u} \quad (23)$$

$$c \ln \frac{q}{u} = c \ln k_j - u \quad (24)$$

$$\ln \frac{q}{u} = \ln k_j - \frac{u}{c} \quad (25)$$

$$\frac{q}{u} = k_j . e^{-\frac{u}{c}} \quad (26)$$

$$q = u. k_j . e^{-\frac{u}{c}} \quad (27)$$

Underwood Model

Underwood assumed the relationship between speed and density as exponential function. The basic formula for Underwood model can be stated by (28).

$$u = u_{ff} . e^{-\frac{k}{km}} \quad (28)$$

Where u_{ff} = free flow speed and k_m = density on maximum flow condition (capacity). Equation (28) can be expressed in logarithmic natural form as shown in (29) which expresses the relationship between speed and density.

$$\ln u = \ln u_{ff} - \frac{k}{k_m} \quad (29)$$

The flow – density relationship can be formulated using the basic equation (1) and by substituting the equation (3) into (28) as shown in (30) – (31)

$$\frac{q}{k} = u_{ff} . e^{-\frac{k}{km}} \quad (30)$$

$$q = k. u_{ff} . e^{-\frac{k}{km}} \quad (31)$$

Equation (31) expressed the relationship between flow and density. The maximum flow (q_m) is reached as $k=k_m$. The flow speed relationship can be formulated using basic equation (1) and by substituting (9) into (28), as shown in (32) – (35).

$$u = u_{ff} . e^{-\frac{q}{u.km}} \quad (32)$$

$$\ln u = \ln u_{ff} - \frac{q}{u.k_m} \quad (33)$$

$$\frac{q}{u.k_m} = \ln u_{ff} - \ln u \quad (34)$$

$$q = u. k_m . \left(\ln u_{ff} - \ln u\right) \quad (35)$$

The equation (35) expressed the relationship between flow and speed. The maximum flow (q_m) is reached as $u=u_m$. $u=u_m$ is formulated in equation (36) – (39).

$$\frac{\partial q}{\partial u} = k_m\left(\ln u_{ff} - \ln u_m\right) + k_m . u_m \left(-\frac{1}{u_m}\right) = 0 \quad (36)$$

$$k_m\left(\ln u_{ff} - \ln u_m\right) - k_m = 0 \quad (37)$$

$$\left(\ln u_{ff} - \ln u_m\right) = 1 \quad (38)$$

$$u_m = e^{\ln u_{ff} - 1} \quad (39)$$

DATA

The data was taken from Virginia Department of Transportation website. The data is traffic flow data collected from road link I-64 Westbound at the upstream of Hampton Road Bridge Tunnel which is located before the bridge as shown by figure 2. This link has two lanes.

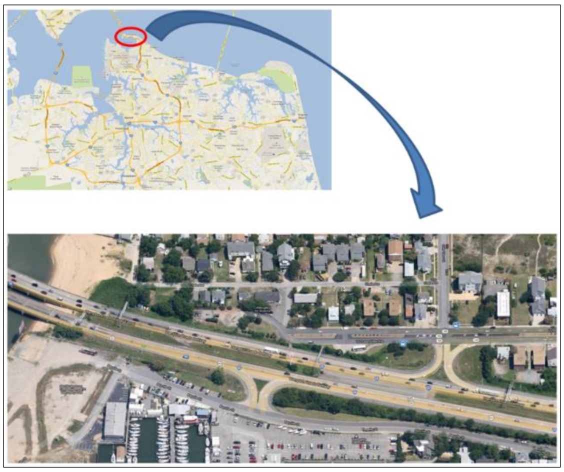


Figure 2. Road link I-64 Westbound

This study utilized twenty four hours traffic data (instead of just taken the AM peak and PM peak) to capture traffic flow characteristic in extreme situations both at free flow speed and traffic jam condition. The date of data collection is September 15, 2011. There are two parameters in the data which are traffic flow (vp15m) and speed (mph). In the data processing the traffic flow data was converted into vehicles per hour (vph) unit.

Calibration of Greenshield, Greenberg and Underwood Model

As has been discussed earlier, Greenshield equation for speed versus density form a linear function,

$$u = u_{ff} - \frac{u_{ff}}{k_j} k$$

Assume that u = Y and k = x, then the equation above can be written in linear equation Y = A + B.x , where $A = u_{ff}$ and $B = -\frac{u_{ff}}{k_j}$. Unlike Greenshields, Greenberg equation for speed versus density form a logarithmic function,

$$u = c \ln\left(\frac{k_j}{k}\right) \quad \Leftrightarrow \quad u = c \ln k_j - c \ln k$$

Assume that u = Y and ln k = x, then the equation above can be written in linear equation Y = A + B.x , where $A = c \ln k_j$ and $B = -c$. In Underwood model, the speed versus density equation form an exponential function,

$$u = u_{ff}.e^{-\frac{k}{k_m}} \quad \Leftrightarrow \quad \ln u = \ln u_{ff} - \frac{k}{k_m}$$

Assume that ln u = Y and k = x, then the equation above can be written in linear equation Y = A + B.x , where $A = \ln u_{ff}$ and $B = -\frac{1}{k_m}$

Traffic flow model

The traffic flow models were formulated using Greenshields, Greenberg and Underwood model. The models were built using regression analyses that were conducted in SPSS. By utilizing the speed and density data, the Greenshields, Greenberg and Underwood model were attained through regression analysis according to the associated function. Table 1 presents result of the regression analysis.

There are at least three relationship obtain from the model, which are speed – density, flow – density and flow – speed relationship. Table 2 presents those relationships.

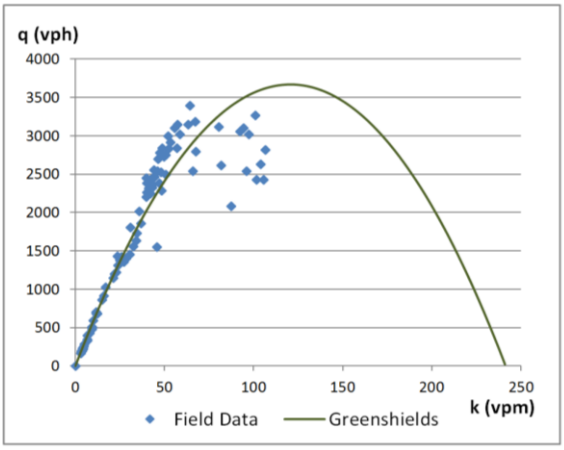
Table 1. Regression Analysis Result

	R^2	A	B
Linear	0.559	60.822	-0.252
Logarithmic	0.232	66.022	-4.590
Exponential	0.571	63.690	-0.006

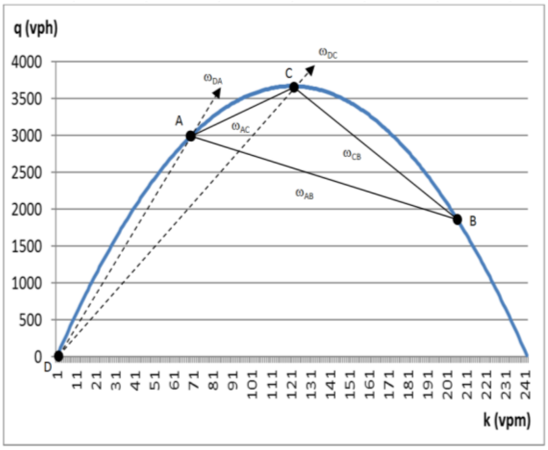
The chosen model was decided by looking at the coefficient determination (R^2) of each model. As shown in table 1 the highest R^2 value is the exponential that represent Underwood model. However, beside the R^2 one would need to evaluate the characteristic offered by this model based on the fact in the field. This model would not work accurately when there is traffic jam condition, since the u-k curve never intersect with x-axis which is the density, thus the jam density (k_j) would never be identified. Hence, by this consideration the chosen model in this case is Greenshields model. The plot of flow and density based on field data versus the one generated from Greenshields equation is presented by figure 3.a.

Table 2. The relationship between traffic flow parameters

	Speed - Density	Flow - Density	Flow - Speed
Greenshields	$u = 60.822 - 0.252\,k$	$q = 60.822\,k - 0.252\,k^2$	$q = 241.357\,u - 3.968\,u^2$
Greenberg	$u = 66.022 - 4.59 \ln k$	$q = 66.022\,k - 4.59\,k \ln k$	$q = 1765383\,u.e^{-0.218\,u}$
Underwood	$\ln u = 4.154 - 0.006\,k$	$q = 63.69\,k\,.e^{-0.006\,k}$	$q = 692.338\,u - 166.667\,u \ln u$



3.a. Field Data vs Greenshield q-k plot



3.b. Shockwave generated by lane closure

Figure 2. Greenshields q-k Diagram

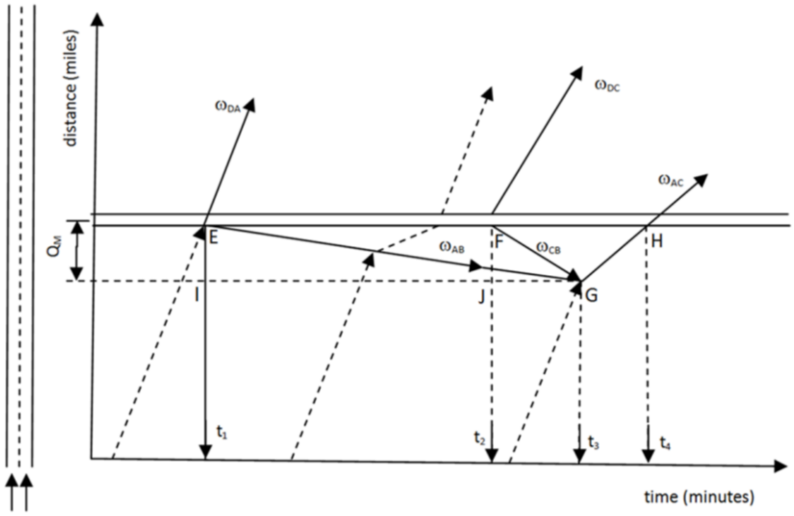


Figure 3. Time and space diagram

Incident Scenario

The shockwave in this study was analyzed using the following scenario; the incident happened when the traffic flow is contant at 3000 vph, 1 lane (out of 2) was closed due to incident. The link capacity is derived from Greenshield model which is 3670 vph, while the capacity of the link was generated from VISSIM simulation. The reason for doing that is, later the result from shockwave analysis will be compared to the VISSIM simulation result, thus to have a fair comparison the input should be identical.

SHOCKWAVE ANALYSIS

Shockwave diagram

Figure 3.b. shows the shockwave diagram as a result of lane closure. Point A showing the flow and density at the moment the incident happened, due to the incident the flow decrease to one lane capacity (point B). Point C shows the flow and density at link capacity (3670 vph).

Shockwave speed and time – space diagram

There are 3 shockwaves in this case (see figure 3). The shockwaves speed are calculated as follows

$$\omega_{AB} = \frac{q_B - q_A}{k_B - k_A}$$

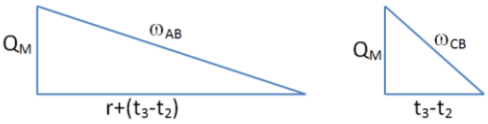
$$\omega_{CB} = \frac{q_B - q_C}{k_B - k_C}$$

$$\omega_{AC} = \frac{q_C - q_A}{k_C - k_A}$$

Using the shockwave diagram, the time - space diagram can be built as shown in figure 4.

t_2-t_1 shows the duration of incident, t_3-t_2 is associated with the total time from lane opening to the time of last vehicle joining the queue, t_4-t_2 is the total time from lane opening to normal condition and Q_M is the queue length. The total delay is the multiplication of EFG triangle area with density value associated with it and the multiplication FHG triangle area with density value related to it.

Total time from lane opening to the time of last vehicle entering the queue



If r = effective duration of lane closure (t_2-t_1), than the t_3-t_2 can be calculated as follow (see Δ EGI and Δ FGJ),

$$\omega_{CB} = \frac{Q_M}{t_3 - t_2} \qquad ;\omega_{AB} = \frac{Q_M}{r+(t_3 - t_2)}$$

$$Q_M = \omega_{CB}(t_3 - t_2); Q_M = \omega_{AB}\big(r + (t_3 - t_2)\big)$$

$$Q_M = Q_M$$

$$\omega_{CB}(t_3 - t_2) = \omega_{AB}\big(r + (t_3 - t_2)\big)$$

$$\omega_{CB}(t_3 - t_2) = \omega_{AB}.r + \omega_{AB}(t_3 - t_2)$$

$$\omega_{CB} = \frac{\omega_{AB}(r)}{(t_3 - t_2)} + \omega_{AB}$$

$$\omega_{CB} - \omega_{AB} = \frac{\omega_{AB}.r}{t_3 - t_2}$$

$$t_3 - t_2 = \frac{\omega_{AB}.r}{\omega_{CB} - \omega_{AB}}$$

$$t_3 - t_2 = \frac{r}{60}\frac{|\omega_{AB}|}{|\omega_{CB}| - |\omega_{AB}|} * 60$$

Length of the queue (Q_M)

The queue length is calculated as follows,

$$Q_M = \omega_{CB}.t_3 - t_2$$

$$Q_M = \frac{r}{60}\frac{|\omega_{CB}|.|\omega_{AB}|}{|\omega_{CB}| - |\omega_{AB}|}$$

Total time from lane opening to normal condition

Using t_3-t_2 and Q_M the total time from lane opening to normal condition is formulated as follow,

$$t_4 - t_3 = \frac{Q_M}{\omega_{AC}}$$

$$t_4 - t_2 = (t_4 - t_3) + (t_3 - t_2)$$

$$t_4 - t_2 = \frac{Q_M}{\omega_{AC}} + (t_3 - t_2)$$

Total Delay

Total Delay is the total area of EGH triangle with density, in this case total delay is multiplication of the area of triangle EFG and its density plus multiplication of the area of triangle FHG and its density

$$TD = \frac{r \,.\, Q_M}{2}(k_B - k_A) + \frac{(t_4 - t_2)\,.\,Q_M}{2} + (k_C - k_A)$$

Shockwaves in various scenarios

Assumed that the arrival rate is 3000 vph, using the same procedure the shockwaves for 10 minutes lane closure are presented in table 3.

Table 3 Shockwave for 1 lane closure

Closed Lane	q_A	k_A	u_A	q_B	k_B	u_B	w_{AB}	q_C	k_C	u_C	w_{CB}	w_{AC}	r	t3 - t2	Q_M	t4 - t2	Total Delay	Total Delay
	vph	vpm	mph	vph	vpm	mph	mph	vph	vpm	mph	mph	mph	minute	minute	mile	minute	veh.min	veh.hr
1	3000	68.96	43.44	1823.00	206.58	8.76	-8.55	3670	115.45	31.73	-20.27	14.41	10	7.30	2.47	7.47	2125.09	35.42
1	3000	68.96	43.44	1823.00	206.58	8.76	-8.55	3670	115.45	31.73	-20.27	14.41	15	10.95	3.70	11.21	4781.46	79.69
1	3000	68.96	43.44	1823.00	206.58	8.76	-8.55	3670	115.45	31.73	-20.27	14.41	20	14.60	4.93	14.94	8500.37	141.67
1	3000	68.96	43.44	1823.00	206.58	8.76	-8.55	3670	115.45	31.73	-20.27	14.41	25	18.25	6.16	18.68	13281.83	221.36
1	3000	68.96	43.44	1823.00	206.58	8.76	-8.55	3670	115.45	31.73	-20.27	14.41	30	21.90	7.40	22.41	19125.83	318.76
1	3000	68.96	43.44	1823.00	206.58	8.76	-8.55	3670	115.45	31.73	-20.27	14.41	35	25.55	8.63	26.15	26032.38	433.87
1	3000	68.96	43.44	1823.00	206.58	8.76	-8.55	3670	115.45	31.73	-20.27	14.41	40	29.20	9.86	29.88	34001.47	566.69
1	3000	68.96	43.44	1823.00	206.58	8.76	-8.55	3670	115.45	31.73	-20.27	14.41	45	32.85	11.10	33.62	43033.12	717.22
1	3000	68.96	43.44	1823.00	206.58	8.76	-8.55	3670	115.45	31.73	-20.27	14.41	50	36.50	12.33	37.35	53127.30	885.46
1	3000	68.96	43.44	1823.00	206.58	8.76	-8.55	3671	115.84	31.63	-20.37	14.31	55	39.82	13.52	40.76	64064.67	1067.74
1	3000	68.96	43.44	1823.00	206.58	8.76	-8.55	3672	116.25	31.53	-20.47	14.21	60	43.06	14.69	44.09	75964.04	1266.07

Result Interpretation of Macroscopic Shock-waves Analysis

By taking the Δ (delta) of $t_3 - t_2$, Q_M and $t_4 - t_2$, it can be concluded that every 5 minutes addition of incident duration will increase,

- the total time from lane opening to the time of last vehicle entering the queue for 3.65 minutes,
- the queue length for 1.23 miles, and
- the total time from lane opening to normal condition for 3.74 minutes

While the total delay increase exponentially (in the power of 3) as it is resulted of the multiplication of area and density.

Modeling using VISSIM

After get the result from macroscopic shockwave analysis, a shockwave simulation using transportation software "VISSIM" was built. The aim of this simulation is two compare the result of macroscopic shockwave analysis, particularly the queue length, with the shockwave resulted from microscopic traffic simulation. The data input for this simulation is network and demand. The network is a single link with two lanes, the link length is 99562.455 ft (18.86 miles), while the demand is 3000 vph and the simulation period is 3 hours.

The incident was created at downstream by installing a traffic signal in one of the lanes. The type of the signal control is fixed time and signal sequence is just red and green. The incident introduced when the traffic light turn to red from green which is setup to give enough time for the vehicles to reach the incident location. At the same place with traffic light, another VISSIM tools was fixed which is queue counters. The queue counters count the queue length (in ft) upstream from the location of traffic signal.

Calibration

It is unlikely that the result of VISSIM simulation will be the same with the of macroscopic shockwave analysis at first attempt, thus some calibrations need to take place. First thing that need to be calibrated is the capacity of a single lane next to the closed lane. By fixing data collection at associated point, the capacity of a link can be attained. The capacity of the single lane is 1823 vph which is the average of 20 times multirun. This number replaced the prior single lane capacity (q_B) in macroscopic shockwave analysis (which in the beginning was assumed to be a half of two lanes capacity).

Next step is calibrating the parameter for queue measurement VISSIM which is the queue definition. The queue definition for this simulation follows the following rule; vehicle will be defined as queuing vehicle when its speed is less than 5.0 mph and this situation end when its speed is greater than 7.0 mph. Other parameter needed to be calibrated in VISSIM is vehicle composition. There are three component of this parameter that affected the result significantly which are vehicle type, composition and desired speed. The vehicle types for this simulation are car, HGV (truck) and bus, which was assigned based on the situation in the field. The desired speed was assigned based on the speed of macroscopic shockwave analysis (43.44 mph at the flow = 3000 vph) which is 42.3 to 48.5 mph for car, 36.0 to 42.3 mph for both HGV and bus, while the relative flow (percentage of each type of vehicle) were assigned using trial and error method to find the optimal composition. The compositions of vehicle in this simulation are distributed as follows, 87% vehicles are car, 15% vehicles are HGV (truck) and 3% vehicles are bus.

Comparison Result

In order to have representative data, each incident duration was simulated for 10 times using the multirun simulation feature in VISSIM. As mentioned before the expected output is queue length. The result is summarized in table 4.

Table 4The Queue Length Comparison

Incident Duration	Q_M (VISSIM)		Q_M Macroscopic	%dif (Q_M vs Average)
	max	average		
10	2.78	2.16	2.47	0.12
15	4.59	3.41	3.70	0.08
20	5.87	4.32	4.93	0.12
25	7.19	5.57	6.16	0.10
30	8.20	6.54	7.40	0.12
35	9.40	7.69	8.63	0.11
40	10.61	8.98	9.86	0.09
45	12.42	10.18	11.10	0.08
50	12.98	11.13	12.33	0.10
55	14.32	12.24	13.52	0.09
60	15.49	13.37	14.69	0.09

Table 4 also shows the comparison of queue length resulted from macroscopic shockwave analysis and the one resulted from traffic simulation in VISSIM. The maximum percentage difference between Q_M average from 10 simulation runs and Q_M Macroscopic is 12% while the minimum is 8% which shows a good match between the two analyses.

CONCLUSION

Using the field data, the Greenshields, Greenberg and Underwood model have been calibrated in this study. In this particular location of study the Underwood model is the best model that fit the data however this model cannot deal with the traffic jam situation, thus for shockwaves analysis the Greenshields model is considered as the best that represent the field data. The shockwave analysis was built using the Greenshields fundamental diagram have shown that the queue length, the total time from lane opening to the time of last vehicle entering the queue and the total time from lane opening to normal condition will increase linearly as the incident duration increase, while the total delay will increase exponentially. Through microscopic simulation established in VISSIM, the queue length also increase as the incident duration increase however the increase is not linear. In this study, the microscopic model has been calibrated successfully to meet the result of macroscopic shockwave analysis, particularly the queue length, and the important calibration factors in this study are speed distribution and vehicle composition.

REFERENCES

1. Papacostas, C.S. and P.D. Prevedouros, *Transportation Engineering & Planning*. Third Edition ed2001: Pearson Education Inc. New Jersey, USA.
2. Immers, L.H. and S. Logghe *Traffic Flow Theory*. 2002.
3. Garber, N.J. and L.A. Hoel, *Traffic and Highway Engineering*. Second Edition ed1999, United States of America: PWS Publishing, A division of International Thomson Publishing.
4. Greenberg, H., *An Analysis of Traffic Flow Theory*. Ops. Res., 1958. **7**: p. 79-85.